ON THE ORIGIN OF COSMIC MAGNETIC FIELDS

H. ALFVÉN
Royal Institute of Technology, Stockholm
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ABSTRACT

As an alternative to the "self-exciting dynamo," a non-stationary model is discussed. The model is based on an effect which Lindberg has discovered at a plasma experiment. The suggested mechanism is the following: In an original poloidal field, hydrodynamic motion produces a toroidal field. When this has reached a certain strength, it becomes unstable. A kink instability changes the field configuration in such a way that an amplified poloidal magnetic field is produced. The process is then repeated. In order to start the field amplification, a very simple pattern of motion is needed, which may easily be produced both in the interior of planets and stars and in interstellar matter.

I. INTRODUCTION

The earth, the sun, and many stars possess general magnetic fields. It is possible that interstellar clouds are magnetized, that spiral arms have regular magnetic fields, and that galaxies also have general magnetic fields. Even if the views of different authors are still conflicting, it seems reasonably certain that interstellar matter is usually magnetized. This makes it likely that there should be some very general process which produces magnetic fields in fluid bodies as different as the earth's fluid interior, the stars, and interstellar matter. The energy required for magnetization can easily be drawn from the kinetic energy of internal motions, but the difficulty is to find a workable mechanism for the production of magnetic fields.

A central position in the discussion is held by Cowling's theorem (Cowling 1934, 1945). If a body has a poloidal magnetic field (for example, as shown in Fig. 1), the rotation around the axis of a ring of the fluid will produce toroidal components of the field which may be much stronger than the initial field. However, according to Cowling, this amplified field cannot be fed back through any *stationary* and *symmetric* process, so that the initial poloidal field increases in strength.

Elsasser (1946, 1947, 1950, 1956) and Bullard (1949, 1950) have made a very remarkable attempt to find a *quasi-stationary*, *non-symmetric* process ("self-exciting dynamo") which could work in the earth's interior. Recently Herzenberg (1957, 1958) has demonstrated that two rotating conducting spheres imbedded in a conducting medium may act as a self-exciting dynamo under certain conditions. Although these results are very important, one still may doubt whether they represent the final solution. The pattern of motion which must be postulated is rather complicated, and it is not very easy to see how it could be produced. Further, even if such a mechanism may work in the earth's interior, it seems very unlikely that it could also be responsible for the magnetic fields of stars and interstellar matter.

Under such conditions it is desirable also to study *non-stationary* processes. This has been done earlier by Alfvén (1950), Lundquist (1951), Dungey and Loughhead (1954), and Inglis (1955), who discussed the possibility that the earth's magnetic field is amplified by a *kink instability* process. The basic idea is that motions in the fluid body change a certain initial magnetic field in such a way that it becomes unstable and, by a convulsion, goes back to its initial shape but with increased strength. This may happen, for example, if in an initial poloidal field motions produce a twist of the magnetic-field lines, by which the total magnetic energy is increased. When the twist exceeds a certain limit which has been calculated by Lundquist and Dungey-Loughhead, it becomes unstable and produces a "kink" with a closed magnetic flux in it. This "new" flux could be used in order to amplify the initial flux. However, it was not very easy to see in detail how and why this flux should amplify the initial magnetic field.

II. PLASMA RING EXPERIMENT

Since these papers were written, our knowledge of magnetohydrodynamic instabilities has increased very much because of "thermonuclear" research. Of special interest is a plasma experiment made by Lindberg, Witalis, and Jacobsen (1960) which seems to introduce a new element into the discussion of the origin of cosmic magnetic fields. By means of a discharge between cylindrical electrodes a ring-shaped plasma was produced which was shot against a radial magnetic field (Alfvén, Lindberg, and Mitlid 1960; Lindberg, Witalis, and Jacobsen 1960). In this way a magnetized plasma ring is formed. This has a toroidal magnetic field, which is produced by the discharge, and a poloidal magnetic field, which consists of the radial flux which has been trapped in the hole of the plasma ring. It was expected that, at any time, the flux of the poloidal

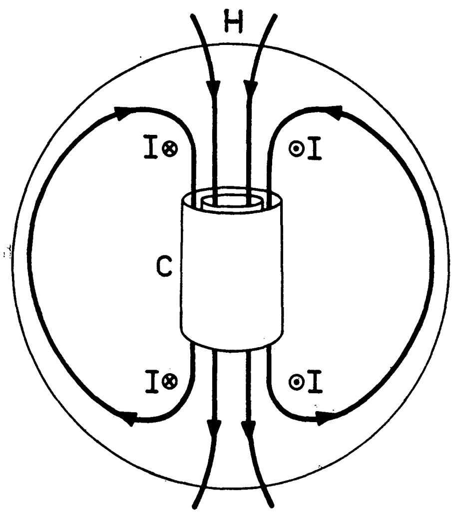

FIG 1.—Simplified picture of the magnetic field H in a fluid sphere. The field derives from electric ring currents I A cylindrical shell C is put into rotation.

field should, at most, equal the initial flux, but it was found by Lindberg that the flux, measured in the established rings, exceeded the initial flux, sometimes by a factor of 5 or even more. As the toroidal field in the experiment is much stronger than the poloidal field, this could be interpreted as an equalization of the fields, so that part of the toroidal field energy is transformed into poloidal field energy. The details of the mechanism are studied by Lindberg and Jacobsen (1961). The essential phenomenon seems to be similar to, although not identical with, the kink formation described in the earlier papers. The current associated with the toroidal field becomes unstable and twisted, so that a helix is formed, and this helical current amplifies the intial poloidal field.

The phenomenon is obviously produced by an instability of the magnetized plasma. There seems to be no reason to believe that it cannot be produced also in an incompressible medium (such as the earth's fluid core).

III. FIELD AMPLIFICATION BY INSTABILITY

According to Cowling's theorem, a toroidal magnetic field cannot amplify a poloidal field by a symmetric and stationary process. The flux amplification observed by Lindberg and Jacobsen is important because it demonstrates that in a conducting fluid a process easily takes place by which a toroidal field amplifies a poloidal field. As this process is not symmetric and stationary, there is no conflict between the experiments and Cowling's theorem.

We shall here outline a mechanism in which this process contributes to the generation of general magnetic fields of celestial bodies. Consider a rotating fluid sphere (angular velocity = ω), which may represent the earth's fluid interior, the sun, an interstellar cloud, or the Galaxy. Let us suppose that its conductivity is infinite and that it is magnetized with a poloidal field with the configuration as shown in Figure 1. We assume that a cylindrical shell with its axis coinciding with the magnetic axis is put into rotation with a linear velocity v. In the case of a celestial body this can be produced by convection, which displaces the cylinder outward or inward a distance Δr. In this case

$$v = 2\omega\Delta r \ . \tag{1}$$

The rotation produces a twist in the magnetic field at both ends of the cylinder, which was put into rotation. In this way a transverse magnetohydrodynamic wave

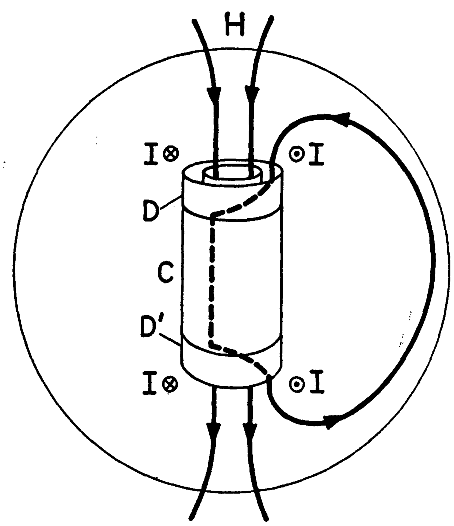

FIG 2.—The rotation of the cylinder C produces a twist of the magnetic field lines, so that in the rings D and D' the magnetic field has a strong toroidal component.

is produced which proceeds with the velocity $V_{MH} = H(4\pi\rho)^{-1/2}$, where H is the initial magnetic field and ρ is the density. Hence, a toroidal magnetic field h is produced in one ring at each end of the cylinder (Fig. 2). The strength of this field is

$$h = H\frac{v}{V_{MH}}, \tag{2}$$

which also means

$$\frac{h^2}{8\pi} = \frac{\rho v^2}{2}. \tag{3}$$

So far, the mechanism is well known. In fact, the production of a toroidal field from a poloidal field is just the process already discussed by Larmor. However, we have to distinguish between two different cases. If the rotation of the cylinder is very slow, a magnetohydrodynamic wave with a small amplitude is produced, which moves out along the field lines. If, on the other hand, the cylinder is put into very rapid rotation, a very strong toroidal field is produced at each end of the cylinder. In this way at each end of the cylinder a magnetic configuration is produced which, according to Lindberg and Jacobsen's experiment, produces an amplification of the poloidal field. The mechanism seems to be that the current $\boldsymbol{j} = \text{curl}\, \boldsymbol{H}$ associated with the toroidal field becomes unstable. The current pattern generating the toroidal field is shown in Figure 3. Because of the instability, the current is transformed into a helix, which leads to an amplification of the poloidal magnetic field (Figs. 4 and 5). Hence, if a process of the same kind as is observed in the plasma experiment also occurs when a similar magnetic configuration is produced in the magnetized celestial body, an amplification of the general magnetic field is produced. In this way an initially weak field can be amplified and sustained.

It is important to find the limit between the case when the rotating cylinder causes only a magnetohydrodynamic wave with a small amplitude and the case when it produces an unstable magnetic configuration. From Lundquist (1951) and Dungey and Loughhead (1954) we know that the condition for instability is that the toroidal field have a strength which is at least of the same order as the poloidal field. If we put, as a condition,

$$h > \alpha H \,, \tag{4}$$

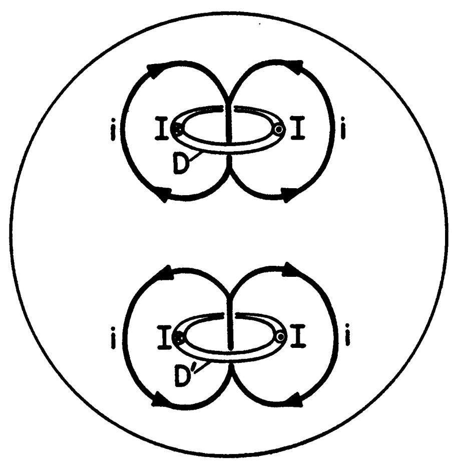

FIG. 3.—Current system. The original poloidal field derives from the ring currents I. The toroidal field in the rings D and D' is associated with the meridional currents i.

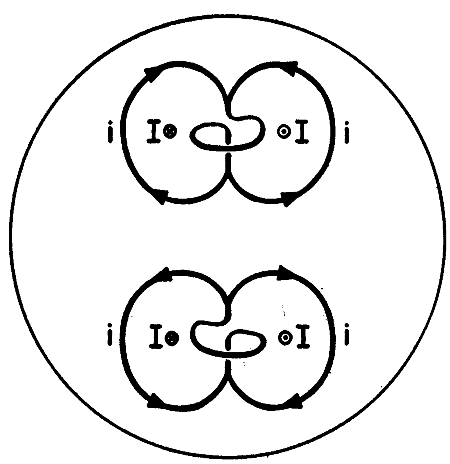

FIG. 4.—At the axis the currents i become unstable for kinks, and two helices are formed, both in such a direction that they tend to increase the poloidal field.

the value of α is known in the simplified cases treated by Lundquist and by Dungey and Loughhead. In general, α is of the order of unity. From equation (2) we find

$$v > \alpha V_{MH} = \alpha \frac{H}{\sqrt{4\pi\rho}}. \tag{5}$$

IV. DYNAMICS OF THE FIELD AMPLIFICATION

The mechanism we are discussing works when in a magnetic field a cylindrical shell is put into rotation relative to the surroundings. Hence the pattern of motion we must postulate is much simpler than what is needed for the "self-exciting dynamo." In the fluid interior of a celestial body such as the earth or the sun, the required motion could be effected by a radial displacement either outward or inward, and this could easily be produced by convection. If the cylindrical shell is abnormally heated (e.g., by release of nuclear energy), an outward displacement is produced which, by our

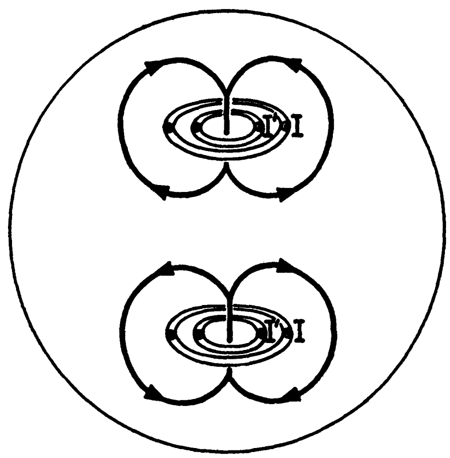

FIG 5.—The helical currents are converted into an axial current and a ring current I', which amplifies the initial current I The configuration is the same as in Fig. 3, but with an amplified poloidal field.

mechanism, results in an increase in the poloidal field. But also if the cylindrical shell is abnormally cool so that it is displaced inward, a similar process is started which also in this case leads to an increase in the poloidal field.

Although the pattern of motion which we have postulated is very simple, it should be considered only as a special case of a much more general type of motion. In fact, any motion which amplifies a magnetic field in such a way that a hydromagnetically unstable configuration is caused may initiate a field amplification. The problem should be studied both experimentally and theoretically.

V. MAXIMUM STRENGTH OF MAGNETIC FIELD

If the differential velocity v of the medium is given and the conductivity is infinite, the amplification of the magnetic field H takes place until H reaches the value given by the inequality (4). Hence our mechanism will be able to amplify a magnetic field up to the strength

$$H \leq \frac{v}{\alpha} \sqrt{(4\pi\rho)} \tag{6}$$

or

$$\frac{H^2}{8\pi} \leq \frac{1}{\alpha^2} \left(\tfrac{1}{2}\rho v^2\right). \tag{7}$$

With $\alpha = 1$, this is a relation which, without very much foundation, has long been used to estimate reasonable values of interstellar fields.

Taking as characteristic values for an interstellar cloud (cf. van de Hulst, 1958) $\rho = 10^{-23}$ gm cm^{-3} and $v = 2 \times 10^5$ cm sec^{-1}, we obtain $H = 2 \times 10^{-6}$ gauss.

For the earth's magnetic field we find that, if this is due to a homogeneous magnetization of the fluid core (radius 3.5×10^8 cm), the field in the core should be $H_c = 3.8$ gauss. As the average density is $\rho = 10$ gm cm^{-3}, we need velocities of only 0.34 cm sec^{-1}. If these velocities are produced by a Coriolis force when a cylindrical shell is displaced radially a distance Δr (cf. Fig. 1), we find from equation (1) $\Delta r = 2.3 \times 10^3$ cm = 23 meters. This very small value of Δr indicates that there are other factors which limit the earth's magnetic field. One of these factors may be the finite conductivity σ. In fact, Elsasser (1946) has suggested that the order of magnitude of the earth's core field should be determined by the relation

$$H = \left(\frac{2\omega\rho}{\sigma}\right)^{1/2} \tag{8}$$

It is not clear whether this result is applicable to our mechanism.

The instability which we have considered need not necessarily occur only at the center of a celestial body, as, for the sake of simplicity, is depicted in the figures. It may take place anywhere in a conducting fluid. As an example, let us suppose that along a magnetic flux tube both the field H and the density vary. At a certain place in the flux tube, torsional magnetohydrodynamic waves are produced which travel along the field lines. We assume that the conductivity is so high that the damping is negligible. When the waves proceed, the induced magnetic field h varies as $\rho^{1/4}$. If in any part of the flux tube $h > \alpha H$, an instability is produced so that the field H is amplified. Hence the process tends to produce a magnetic field H in the tube which is H = const. $\rho^{1/4}$.

Both large-scale convulsions of the whole fluid body and small-scale processes may be important in amplifying and sustaining cosmic magnetic fields.

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