STOCKHOLMS OBSERVATORIUMS ANNALER
(ASTRONOMISKA IAKTTAGELSER OCH UNDERSÖKNINGAR Å STOCKHOLMS OBSERVATORIUM)
BAND 14. N:o 2.

ON THE COSMOGONY OF THE SOLAR SYSTEM

BY

HANNES ALFVÉN

WITH 6 FIGURES IN THE TEXT

COMMUNICATED NOVEMBER 12th 1941 BY C. W. OSEEN AND BERTIL LINDBLAD

STOCKHOLM
ALMQVIST & WIKSELLS BOKTRYCKERI-A.-B.
1942

UPPSALA 1942
ALMQVIST & WIKSELLS BOKTRYCKERI-A.-B.

On the Cosmogony of the Solar System

by

HANNES ALFVÉN.

With 5 figures in the text.

Communicated November 12th 1941 by C. W. OSEEN and BERTIL LINDBLAD.

Abstract.

The motion of charged particles under the combined action of solar gravitation and the sun's magnetic and electric fields is studied. It is found that ions and electrons within the limits of the planetary system are affected very much more by the sun's general magnetic field than by solar gravitation. As at the time of genesis of the planetary system, the matter of which the planets were formed is likely to have been more or less ionized, it is probable that electromagnetic forces have been more important than mechanical forces.

The fact that the main part of the rotational momentum of the solar system belongs to the outer planets and not to the sun has constituted a crucial difficulty for all cosmogonies of the Laplacian type. It is shown that this difficulty is removed if electromagnetic forces are introduced into the theory.

The study of the motion of ions and electrons in the sun's general magnetic field leads to a cosmogony of a new type. The formation of the planets is attributed to the passage of the sun through an interstellar gas cloud. It is shown that a gas cloud approaching the sun will give rise to an accumulation of matter at about that distance from the sun, where the big planets are found.

The theory attempts to account for the main features of the planetary system. It can also explain some details of the structure, e. g. why Saturn alone has rings, and even account for the size of the different rings.

An empirical relation between the orbital radii and the masses of the planets (and the satellites) is found.

A. General principles.

1. Electromagnetic forces compared with mechanical forces.

The discussion of the cosmogony of the planetary system has been dominated by two theories: LAPLACE's theory and the tidal theory. In their present state neither of them is able to account for the main features of the planetary system. The Laplacian theory cannot explain the fact that the main part of the angular momentum in the solar system is possessed by the outer planets and not by the sun. This difficulty is stressed by the fact that it has been shown that the viscosity of the gas supposed to have generated the planetary system is too small to have transferred a considerable momentum from the sun to the outer planets. On the other hand the tidal theory or theories seem to encounter still more objections. The collision between two stars is a very improbable process and the discussion points more and more in the direction that even if such a collision had occurred it could not have resulted in anything similar to our planetary system. We are thus in the very awkward position that the planetary system cannot have come into existence. Eppur si muove!

The conclusion must be that at the genesis some type of force was in action, of which no account has been taken by the present theories. They employ gravitation and other mechanical forces, but they do not introduce electromagnetic forces. However, if the planetary system has originated from a gas cloud, it is very likely that this has been more or less ionized, and if so electromagnetic forces certainly have been at least as important as mechanical forces.

The relative magnitude of mechanical and eloctromagnetic forces within the region of the planetary system can be illustrated by the following example. An ion (mass $= m$, charge $= e$) moves with the velocity $v_\perp$ in the equatorial plane of the sun's magnetic field at a distance r from the sun. It is subject to the solar gravitation

$$f_g = \frac{k M_\odot m}{r^2} \tag{1.1}$$

($k =$ gravitational constant, $M_\odot =$ mass of sun), and to the force from the magnetic field of the sun:

$$F_m = \frac{e}{c} v_\perp H = e\, v_\perp\, a\, r^{-3} \tag{1.2}$$

($a =$ sun's magnetic dipole moment). We put

$$\eta_g = \frac{f_g}{F_m} = \frac{c}{e\,H} \frac{f_g}{v_\perp}. \tag{1.3}$$

For a proton ($m = 1.66 \cdot 10^{-24}$ g, $e/c = 1.60 \cdot 10^{-20}$ e m u) moving with the orbital velocity of the earth ($v_{\perp} = 3 \cdot 10^6$ cm/sec) at the distance of the earth from the sun ($r = 1.5 \cdot 10^{13}$ cm) we find ($k = 6.66 \cdot 10^{-8}$; $a = 4.2 \cdot 10^{33}$ gauss cm^3; $M_{\odot} = 1.98 \cdot 10^{33}$ g).

$$f_g = 0.97 \cdot 10^{-24} \text{ dyn} \tag{1.4}$$

$$F_m = 6.0 \cdot 10^{-20} \text{ dyn} \tag{1.5}$$

$$\eta_g = \frac{1}{60\,000}. \tag{1.6}$$

Thus *the force due to the magnetic field of the sun is 60 000 times as large as the solar gravitation!* Even at the distance of Pluto a proton moving with the same velocity as Pluto is much more affected (250 times) by the magnetic force than by solar gravitation! For electrons η is still smaller, but for heavy ions it is larger (proportional to their mass).

Consequently there seem to be strong indications that at the genesis of the planetary system *electromagnetic forces have been more important than mechanical forces.*

Moreover, in a recent paper(4) it has been shown that if ionized matter is brought into the neighbourhood of the sun, electromagnetic forces will tend to make it take part in the solar rotation. This effect is caused by the polarisation due to the magnetic field of the sun. As is usually the case when a conducting body moves in a magnetic field, a system of currents is produced, which tends to brake the relative motion.

In the cited paper(4) it is shown that the length of time necessary for the sun to transfer a great part of its angular momentum to the ion cloud surrounding it, may be as short as 10^5 years (see § 3 in the cited paper).

As this transfer has been the principal difficulty in the Laplacian theory (and also a difficult point in the tidal theories) we are encouraged to try to revise the cosmogonic hypotheses.

2. On the motion of charged particles.

In the first place we must study the laws of motion of a gas — more or less ionized — in the environment of the sun (especially within the present limits of the planetary system). Let us assume that the gas is very thin so that its total mass is negligible in comparison with the solar mass. If the gas is non-ionized it obeys the ordinary laws of a gas under the influence of (solar) gravitation. If it contains ions and electrons, these are also affected by electric and magnetic fields. Of greatest importance is the effect of the sun's general magnetic field.

The type of motion of charged particles (electrons and ions) depends upon their energy. If this is small enough, we can treat the problem by a simple method outlined in a recent paper (2). The condition for the validity of this method is that the distance from the dipole (moment $= a$) to the particle (mass $= m$, charge $= e$, energy $= E$) is smaller than

$$r = 0.1 \left(\frac{e\,a}{c}\right)^{1/2} (2\,m\,E)^{-1/4} \tag{2. 1}$$

(formula (11.2) in the cited paper(2)). We are especially interested in the case when the energy is of the same order of magnitude as the gravitational energy of the particle in relation to the sun, because this energy is attained if the particle has fallen in towards the sun from interstellar space. Thus, if we put

$$E = \frac{k\,M_\odot\,m}{r} \tag{2. 2}$$

($k =$ gravitational constant, $M_\odot =$ mass of sun) and introduce this into (2. 1) we obtain ($k = 6.66 \cdot 10^{-8}$; $e/c = 1.6 \cdot 10^{-20}$; $M_\odot = 1.98 \cdot 10^{33}$; $m_H = 1.67 \cdot 10^{-24}$; $a = 4.2 \cdot 10^{33}$)

$$r = 85\,.\,10^{13}\,m'^{-2/3} \tag{2. 3}$$

where $m' = \frac{m}{m_H}$ is the atomic weight of the ion.

Consequently, for electrons $\left(m' = \frac{1}{1\,840}\right)$ and for protons ($m' = 1$) the method is applicable within the whole planetary system and even outside the orbit of Pluto ($r = 64 \cdot 10^{13}$ cm). For heavy ions the region of validity as defined by (2. 1) is not so large with the present value of the dipole moment of the sun. For oxygen ions for example we have $r \leq 14 \cdot 10^{13}$ cm which is about as much as the orbital radius of Saturn. However even about five times outside this limit the method gives a rather good approximation. Consequently, there seems to be little danger in applying the method to the motion of charged particles (except very heavy ions) over the whole region of the planetary system.

According to the cited paper(2) a charged particle moves in a small circle around the magnetic lines of force. This circle is subject to a drift. In order to calculate the motion of a particle starting with the velocity v from a given point, we first must calculate the components $v_{||}$ and $v_\perp$ parallel to and perpendicular to the magnetic field. The component $v_\perp$ gives a motion in a circle, the radius of which is small (which is a condition for the applicability of this method). For the spiralling particle an »equivalent magnet» (at the centre of the circle) is substituted, which has the moment

$$\mu = \frac{E_\perp}{H} = \frac{m v_\perp^2}{2H} \tag{2.4}$$

(H = magnetic field). During the motion, μ remains constant.

The small circle in which the particle moves, is subject to a drift which has the components v_x, v_y, v_z. Employing an orthogonal curvilinear coordinate system, the z-axis of which is parallel to the magnetic field H we have

$$v_x = \frac{c}{eH} f_y \tag{2.5}$$

$$v_y = -\frac{c}{eH} f_x \tag{2.6}$$

$$\frac{d}{dt} v_z = \frac{1}{m} f_z. \tag{2.7}$$

Here v_z means the same as $v_{||}$. The force $f = (f_x, f_y, f_z)$ is the sum of all forces acting upon the »*equivalent magnet*». This means that to the real forces acting upon *the particle* (e.g. gravitation and electric field) we must add two fictive forces: The first of them, the magnetic force

$$f^m = -\mu \text{ grad } H \tag{2.8}$$

is due to the repulsion of the equivalent magnet by the central dipole[1]. The second one, which is due to the curvature of the magnetic lines of force, is given by (8. 12) in the cited paper, but is of little interest in the present problems.

We call the plane of a magnetic line of force the y-z-plane (curvilinear orthogonal coordinates). All forces of most importance act in this plane. In fact, these forces are (see § 12): gravitation (from the central body) which is parallel to the vector radius; centrifugal force (if the particles take part — more or less — in solar rotation) which is perpendicular to the axis of the sun; magnetic force (2. 8), which also is situated in the y-z-plane.

Thus, as in (2. 6) f_x cancels, v_y is also zero. This means that *charged particles can move only on the surfaces engendered through rotating the magnetic lines of force around the axis of the sun.*

3. On the ionisation of a gas cloud approaching the sun.

It is of interest to study what happens if a gas cloud enters the solar field. Let us suppose that the cloud consists of neutral as well as ionized atoms having only small

[1] It must be observed that f^m is not the same as F_m in § 1.

velocities at infinite distance from the sun. The ions (and electrons) are attracted by the solar gravitation but they are also acted upon by the solar magnetic field, even at a very large distance from the sun. It is easily seen that the magnetic field makes it impossible for them to reach the present region of the planetary system. Consequently we do not need to take account of them. On the other hand, a neutral atom will fall towards the sun under the action of gravitation alone, and it will move in a parabola. Its kinetic energy at the distance r from the sun is

$$E_g = \frac{k\, M_\odot\, m}{r}. \tag{3.1}$$

If a large number of atoms fall under the influence of solar gravitation, their (small) initial velocities being distributed at random, they will collide mutually, so that their motion becomes irregular. We can speak of a temperature given through the condition that the mean energy of the atoms equals E_g (given by 3. 1). The process is simply the heating of a gas under the action of a gravitational field.

The more the gas approaches the sun, the warmer it becomes. At a certain distance r_i from the sun its thermal energy is large enough to ionize it. If V_{ion} is the ionisation potential of the gas, this occurs when

$$e\, V_{\text{ion}} = E_g = \frac{k\, M_\odot\, m}{r} \tag{3.2}$$

i. e. at the distance

$$r_i = \frac{k\, M_\odot\, m}{e\, V_{\text{ion}}} = 6.9 \cdot 10^{-20} M_\odot \frac{m'}{V_{\text{ion}}} = 13.5 \cdot 10^{13} \frac{m'}{V_{\text{ion}}}\ \text{cm} \tag{3.3}$$

(m' = atomic weight, V_{ion} in volts). Although the gas will be partly ionized already somewhat outside this limit, we schematize the conditions, assuming that the ionisation occurs exactly at the distance r_i.

When the gas, having reached the distance r_i from the sun, becomes ionized, electromagnetic forces come into action. Of main importance is the force f^m defined by (2. 8), which gives a (magnetic) repulsion of the particle from the sun. Leaving the details of the phenomenon to be discussed later (in § 12), we shall only state here that the repulsion f^m is usually larger than the gravitational attraction. This means that *when the gas has been ionized it is repelled by the sun.* Consequently, *a neutral gas falling in towards the sun from infinity is stopped at about the distance* r_i (given by 3. 3) *from the sun.*

If the sun on its way through space passes a gas cloud, the gas it attracted by gravitation towards the sun as long as it is outside the distance r_i. However inside the spherical surface r_i, at which the gas becomes ionized, it is repelled by the magnetic field. The result must be that *the gas is accumulated at about the distance r_i from the sun.* If the gas condenses to planets, we must expect that they, or at least the most heavy of them, are situated at about the distance r_i. In our planetary system the main part of the mass is concentrated in Jupiter and Saturn. Is it possible to explain this through assuming that r_i equals the distance to Jupiter or Saturn?

The gas cloud supposed to approach towards the sun is likely to be a mixture of those elements which are most abundant in the universe. These are supposed to be *H*, *He*, *C*, *O*, *N*, *Na*, *Mg*, *Si*. Their ionisation potentials are, in volts: *H*: 13.5; *He*: 24.5; *C*: 11.2; *O*: 13.6; *N*: 14.5; *Na*: 5.1; *Mg*: 7.6; *Si*: 8.1. We put $V_{\text{ion}} = 12$ volts as a rough weighted mean of their ionisation potentials. If we put $r_i = 7.78 \cdot 10^{13}$ cm (mean distance of Jupiter) we obtain from (3.3)

$$m' \approx 7$$

(For Saturn we obtain $m' \approx 13$.) Consequently, *if the gas has an average atomic weight* $= 7$ (and an average ionisation potential $= 12$ volts) *it is stopped at the distance of Jupiter.*[1] This value of the average atomic weight seems to be a very reasonable figure for a gas cloud constituted of the elements most abundant in the universe. Therefore, *it is quite likely that our gas cloud will be concentrated at about the distance from the sun of the heavy planets.*

Summary of A.

If a cloud of ionized gas is at rest in the neighbourhood of the rotating sun, electromagnetic induction due to the sun's general magnetic field will produce a system of currents. These currents give rise to forces which tend to accelerate the ion cloud and retard the solar rotation, thus equalizing the angular velocities. The forces may be so large that they can transfer a considerable part of the rotational moment of the sun to the cloud in a rather short time.

It is expected theoretically that the retardation of the solar rotation should be especially great at high heliographic latitudes. The fact that the solar rotation is slower near the poles indicates that a process of this kind has been in action.

If so, we must expect a considerable part — perhaps the main part — of the rotational moment of the sun to have been transferred to the ion cloud. It is well known that the main part of the rotational moment of the solar system does not belong to the sun — as would be expected according to practically all cosmogonic theories —

[1] Because of a correction discussed in § 12, the value of m' must be increased by 50 % to $m' = 10$.

but to the outer planets. Consequently it is reasonable to suppose that our hypothetical ion cloud, after having taken up most of the rotational moment of the sun, has condensed to planets. In this way the non-uniform solar rotation and the remarkable distribution of angular momentum in the solar system is explained by the same process.

Thus we are led to suppose that the planets have been formed out of an *ionized* cloud. It is shown that *ions* — even at the distance of Pluto from the sun — *are affected very much more by the magnetic field of the sun than by solar gravitation.* This means that at the genesis of the planetary system electromagnetic forces have been more important than mechanical forces. The introduction of electromagnetic forces solves some of the difficulties encountered in purely mechanical cosmogonies.

The cloud which was to form the planets is likely to have arrived in the environment of the sun from interstellar space: the sun has passed a nebula. However if the cloud has been ionized initially it is repelled by the magnetic field of the sun and cannot approach even to the distance of Pluto. On the other hand, if the cloud is non-ionized initially it can approach and in doing so it becomes heated and consequently ionized. The motion of the cloud is studied and it is found that the matter becomes accumulated at about that distance from the sun where the big planets are situated.

B. Mass distribution in the solar system.

4. The outer planets.

Let us discuss a little more closely the process outlined in § 3. Suppose that the atoms of the gas cloud fall in towards the sun in the same number from all directions. Then the atoms arriving on the sphere r_i during a short interval of time will give a uniform density over the whole spherical surface. After their ionisation they can only move along the magnetic lines of force according to § 2 (rotation around the axis of the sun is also allowed). A stable equilibrium is possible only at the point where the magnetic line of force cuts the (magnetic) equatorial plane, so that — sooner or later — every ionized particle must reach this plane. (See § 12). Consequently our gas must be condensed in the equatorial plane, where the density of it will be that which is obtained through projecting along the magnetic lines of force a spherical surface with constant density. (See Fig. 1). Let us calculate this density in the equatorial plane.

If the mass distribution is uniform over the surface of the sphere r_i, the mass dM between the parallel circles of the latitudes φ and $\varphi + d\varphi$ amounts to

$$d M = M_0 \cos \varphi \, d \varphi \tag{4.1}$$

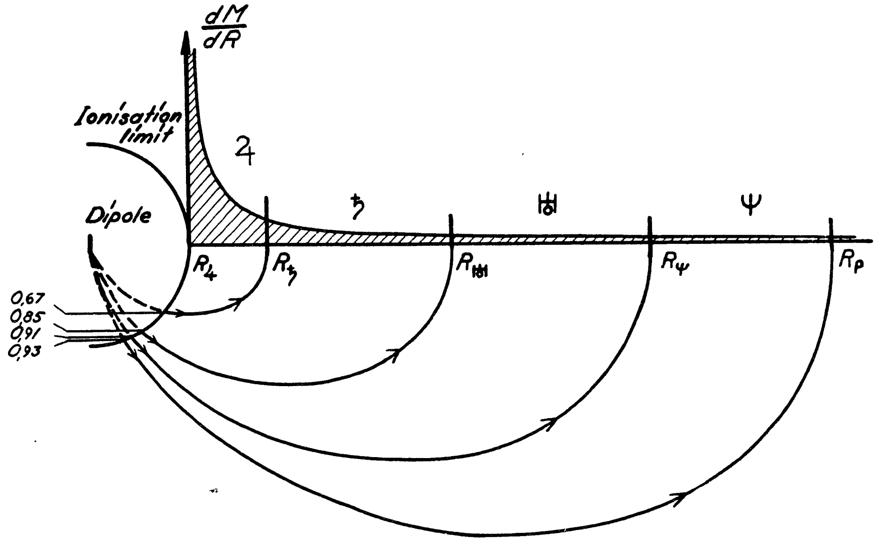

Fig. 1. Neutral atoms arriving from space are ionized at the spherical surface marked »Ionisation limit». From this surface the ions move along the magnetic lines of force to the equatorial plane where their density $\frac{dM}{dR}$ becomes as shown. The figures to the left mark the fractions of the spherical surface.

(M_0 = constant). The equation of the magnetic lines of force is

$$r = R \cos^2 \varphi \tag{4.2}$$

where r is the distance to sun from a point on the line of force, and R the point where it cuts the equatorial plane. Putting $r = r_i$ we obtain

$$d\varphi = \frac{1}{2} \frac{\cot \varphi}{R} dR \tag{4.3}$$

and

$$\frac{dM}{dR} = \frac{M_0 r_i}{2} \frac{1}{R^2 \sqrt{1 - \frac{r_i}{R}}} \tag{4.4}$$

This function is plotted in Fig. 1.

Let us now see if it is possible that the outer planets have originated from a gas having the density given by (4.4).

Without discussing the mechanism of condensation in detail, we assume roughly that r_i coincides with the present value of the orbital radius of Jupiter ($R_♃$) and that all gas situated between $R_♃$ and the orbital radius of Saturn ($R_♄$) is used to build up Jupiter[1]. In the same way we assume that all matter between $R_♄$ and $R_♅$ (Uranus) is condensed to Saturn, etc. Thus we should expect the following masses of the planets:

[1] That according to § 13 all distances are likely to decrease by a factor of 2/3 does not matter in this respect.

Jupiter

$$m_{♃} = \varkappa \int_{R_{♃}}^{R_{♄}} \frac{dR}{R^2 \sqrt{1 - r_i/R}}. \tag{4.5}$$

Neptune

$$m_{♆} = \varkappa \int_{R_{♆}}^{R_p} \frac{dR}{R^2 \sqrt{1 - r_i/R}}$$

(R_p is the orbital radius of Pluto, $\varkappa$ is a constant).

The relative masses of the planets calculated from (4.5) and those really observed are given in the following table. The calculated mass of Jupiter has been put equal to the observed mass.

Planet	Mass (Earth = 1) Calculated	Observed
Jupiter	317	
Saturn	87	95
Uranus	26	15
Neptune	10	17

The calculated values agree with observations within a factor of 2. The integral from Pluto to infinity gives 32 units.

The assumption that the gas is divided exactly at the present distances of the planets is of course somewhat arbitrary. But if we go in the opposite direction we can interpret the result as follows. Suppose that we »smoothe out» the masses of the outer planets so that we obtain a continuous mass distribution in the equatorial plane A projection of this along the magnetic lines of force upon a sphere gives us an almost uniform mass distribution. Consequently our theory is reconcilable with the mass distribution observed in the planetary system.

5. The inner satellite systems.

If we are on the right way in our interpretation of the planetary system, we must be able to apply the same considerations to the satellite systems. Suppose that after the formation of the planets a small fraction of the gas cloud is still left. This gas falls in towards the planets. As the earth as well as the sun are magnetic dipoles it is very likely that all celestial bodies are magnets. If so a gas falling down towards a planet and consequently heated and ionized will be stopped by the magnetic field of the planet in the same way as discussed above. The ionisation occurs at the distance

$$r_i = \frac{k M_p m}{e V_{\text{ion}}}$$

where M_p is the mass of the planet. This means that *the gas must be accumulated in a region where the gravitational energy M_p/r in relation to the planet has the same value as the gravitational energy $M_{\odot}/r$ (in relation to the sun) for the heavy planets.*

Fig. 2 shows the gravitational energies M_p/r of the satellites of the outer planets plotted in the same diagram as the gravitational energies of the planets. It is very remarkable that *the four big satellites of Jupiter and the inner satellites of Saturn have the same gravitational energy in relation to Jupiter and Saturn as the heavy planets have in relation to the sun*, just as expected. Thus, if our mechanism has formed the outer planets it has also been able to form these satellite systems.

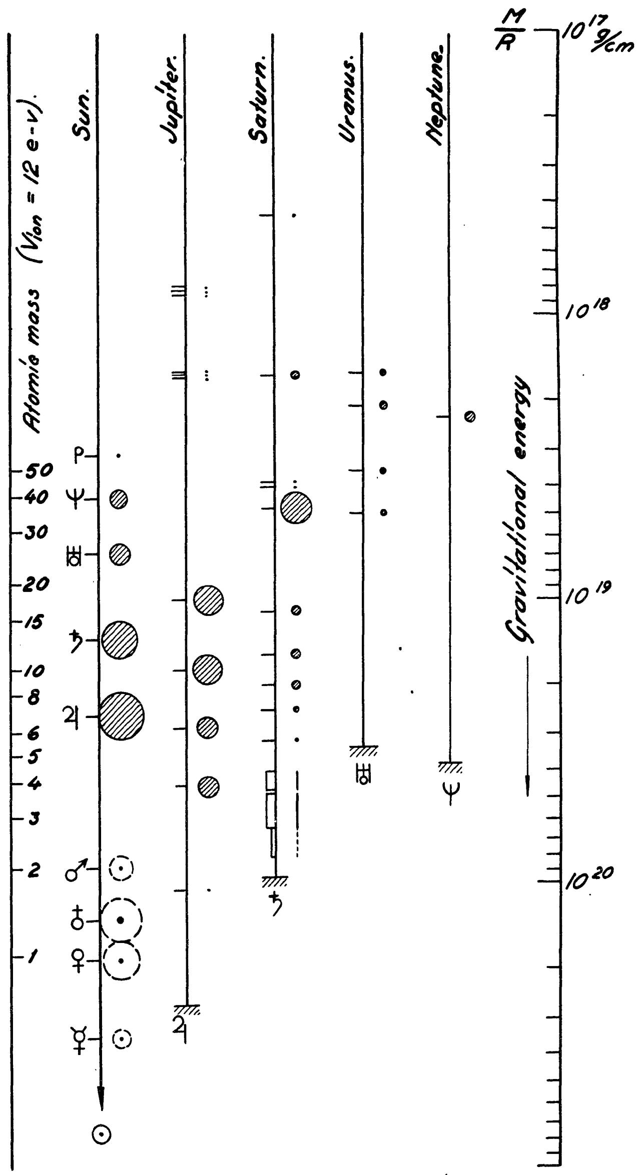

Fig. 2. Gravitational energy of planets and satellites.

In the Jovian system the four big satellites have about the same gravitational energy as the outer planets. The value of $\frac{M_{\odot}}{r}$ for the innermost of the outer planets (= Jupiter) amounts to $25 \cdot 10^{18}$ g/cm. In § 3. we assumed that the invading gas becomes ionized at this value and is stopped. If the gas approaching Jupiter has been ionized and stopped when exactly the same value of M/r is reached, we should expect the value of $\frac{M_{♃}}{r}$ for the first Jovian satellite to be the same. In reality it is $45 \cdot 10^{18}$ g/cm. If significance is attributed to this discrepancy (by a factor of $\frac{45}{25} = 1.8\Big)$ it could be accounted for by the assumption that the mass of Jupiter has increased since the ionisation of the gas, which later has built up the first satellite. It is easily seen that the increase by a factor of only $\sqrt{1.8} = 1.34$ is enough to account for the discrepancy.

An alternative explanation is that the mean atomic weight of the gas has decreased by a factor of 1.8 since the formation of the planets started. This would be natural because the heaviest components of the gas may be expected to condense at first. If the decrease in atomic weight goes on continuously, the ionisation sphere r_i shrinks gradually, and satellites are formed closer the planet. This process would also explain why the masses of the satellites do not obey the same simple law as the outer planets.

In the Saturnian system the five inner satellites and the rings have about the same values of M/r as the critical values in the solar and in the Jovian systems. (If we identify the inner limit of the rings with r_i, the mass of Saturn must have increased by a factor of nearly 2 since the gas invasion began or alternatively the atomic weight must have decreased by a factor of almost 4.) It is well known that a satellite within the Roche limit must break up into a dust ring. This means also that if matter is brought within this limit it will form dust rings rather than the planets or satellites which we have assumed that it usually forms. Now, because Saturn is much smaller than Jupiter, *the critical region* (where the invading gas is ionized and stopped) *lies much closer to its surface, indeed below the Roche limit.* Consequently, mass is accumulated below the Roche limit, which means that dust rings must be formed.

Uranus and Neptune are still smaller than Saturn. At their surfaces the values of $\frac{M}{r}$ amount to $34 \cdot 10^{18}$ and $38 \cdot 10^{18}$. These values are close to the critical values ($25 \cdot 10^{18}$ for the solar system and $45 \cdot 10^{18}$ for the Jovian system). Consequently, neutral gas falling down upon them does not become ionized until very close to their surfaces, or is not ionized at all before it hits the surface. As is later shown in a closer study, the stopping does not take place exactly at the ionisation distance, but somewhat below, so that even if the ionisation limit lies a little above the surface, *all the invading gas will hit the surface.* Consequently, *no satellites or rings can be formed by this process.* No satellites with $\frac{M}{r}$-values in the same region as the planets and satellites discussed have actually been found. (The satellites of these planets have very low M/r-values and must have been formed by some other process. See § 7.)

Thus, we are in the position to be able to answer the old question: *why has Saturn* *rings, and why Saturn alone?* The reply is: Saturn has a mass just so large that $\frac{M_{\text{planet}}}{r_{\text{sat}}}$ attains the critical value above its surface but below the Roche limit. If the mass were larger — as Jupiter's —, the invading gas would have been stopped above the Roche limit, forming satellites, but no rings. If the mass had been smaller, as in the case of Uranus and Neptune, the invading gas would have struck the planetary surface before being stopped, and could have formed neither satellites nor rings.

6. Natural groups of bodies in the solar system.

We have followed a simple process, the invasion and ionisation of a gas cloud, and found that in this way we have some chance of accounting for the formation of two groups of bodies:

1. The outer planets (Jupiter-Neptune).
2. The inner satellites (i. e. the four big Jovian satellites and the inner satellites — Mimas to Rhea — and the ring system of Saturn).

The other members of our solar system are:

3. The inner planets, including the asteroids.
4. The outer satellites: the satellites Titan-Iapetus of Saturn, and the four satellites of Uranus.
5. Abnormal satellites: the moon, the sixth to ninth Jovian, the ninth Saturnian, and the Neptunian satellites.
6. Some very small bodies (Martian satellites, fifth Jovian satellite).
7. The comets.

As the mass of the members of the sixth and seventh groups are very small they are not of primary interest to our analyzis, which in the first place tries to account for the mass distribution in the solar system.

The fifth group must be accounted for by special assumptions. The earth and the moon form a sort of double planet. The abnormal satellites of Jupiter and Saturn are probably captured asteroids, as has been pointed out by several authors. The Neptunian satellite has a retrograde rotation.

Consequently, the problem we have to attack at first is that of the formation of the third and the fourth groups.

7. The outer satellites.

As shown by Fig. 2 the outer satellites of Saturn and the four satellites of Uranus have the same gravitational energy. Indeed, the lower as well as the upper limits of the two systems almost exactly coincide. The value of $10^{-18} \frac{M}{R}$ equals 4.77 for Titan and 4.95 for Ariel, the innermost satellites of the systems. For the outermost satellites it amounts to 1.63 for Iapetus and 1.62 for Oberon. (The ninth Saturnian satellite, Phoebe, with $10^{-18} \frac{M}{R} = 0.45$ is abnormal, probably a captured asteroid as indicated by its retrograde motion and high excentricity.)

It is possible that the fourth group, the outer satellites, is generated in the same way as the outer planets and the inner satellites. The only change we need to make in our assumptions is that the value of m'/V_{ion} of the invading gas is much larger, so that the gas is stopped at a smaller value of M/r. If we put $M/r = 4.9 \cdot 10^{18}$ (the value of Titan and of Ariel, we obtain $m'/V_{\text{ion}} = 3.4$). If we assume, as has earlier been done, that the mean value of V_{ion} is 12 volts, the mean atomic weight must be as high as 41. However this value is probably too high, because most heavy atoms have a value of V_{ion}, which is smaller than 12 volts. This means that the mean atomic weight is reduced, perhaps to about 30 (if V_{ion} is put equal to 9 volts).

If the outer satellites are generated by a heavier gas than the inner satellites we should expect them to have a higher density. The only outer satellite of which the density is known, is Titan, which has the value 3.5 (1). The inner satellites of Saturn have very low densities (probably less than 1), and the big Jovian satellites have the densities 2.9, 2.9, 2.2 and 0.6. Consequently, if the cited values are reliable, Titan has a higher density than any known density of an inner satellite, which is in agreement with our expectations.

At the first examination, we should expect that the process discussed above must give »outer satellite» systems to Jupiter and Neptune as well as to Saturn and Uranus. However, there seem to be good resons why it has not.

In the case of Jupiter, the gravitational field of the sun is of some importance in the region where the »outer satellite» system might be expected. In fact the solar gravitation equals the Jovian gravitation at the distance $r_{♃} = 2.4 \cdot 10^{12}$ cm, which corresponds to $10^{-18}\ M/r = 0.8$. Certainly this is outside the outer satellite region, but the disturbance may be of importance even closer to the planet. (For Saturn the solar and planetary gravitations are equal at a distance $r = 2.4 \cdot 10^{12}$ cm, corresponding to $10^{-18} m/r = 0.23$, which is much outside the outer satellite region).

It is also possible that the solar magnetic field has disturbed the production of outer satellites around Jupiter. We do not know the actual strength of the Jovian magnetic field, still less its magnitude at the genesis of the satellite systems.

Neptune has an »abnormal satellite» (retrograde motion, exceptionally large mass) in that very region where the »outer satellite» system is expected. It is probable that disturbances from it have prevented the building up of an ordinary system.

8. The inner planets.

The inner planets can certainly not have been generated by a process of the same kind as that outlined above. A gas coming from outside the solar system cannot pass the orbit of Jupiter unless it has a lower value of m'/V_{ion} than that gas which has formed the outer planets. But if it has, its mean atomic weight must be low so that the densities of the inner planets must be lower instead of higher than that of the outer planets. Further, not even if the gas were pure hydrogen, for which the value of m'/V_{ion} has its lowest value ($= 0.075$), the gas could reach closer to the sun than $13.5 \cdot 0.075 \cdot 10^{13} = 1.0 \cdot 10^{13}$ cm (according to 3.3). This is about the orbital distance of Venus, but still 1.7 times the distance of Mercury.

It is possible that the inner planets have been formed out of gas not coming from interstellar space, but expelled from the sun. However a theory along this line would meet all the difficulties encountered in those theories that have tried to explain the whole planetary system in this way.

If the inner planets have been formed neither from gas coming from interstellar space nor from gas expelled from the sun, we have still the possibility that the raw material is *small solid particles*, let us say *meteoric dust*. A cloud of such particles, coming from interstellar space, can reach the region of the inner planets without being stopped by the magnetic field of the sun. In fact, even if a dust particle is charged to a rather high potential, its specific charge (e/m) is usually much smaller than that of an ordinary ion, and consequently it is not very much affected by the magnetic field of the sun.

The high density of the inner planets makes it possible that the raw material could be a meteoric dust cloud. How it is transformed into planets is of course a very difficult question. The similarity in many respects (orbits, axial revolution, relative spacings) between the outer and the inner planets indicates that the formation processes must have been of about the same kind. Because of that it seems most reasonable to assume that when the meteoric dust has reached the neighbourhood of the sun, or say the region of the inner planets, it is volatilized in some way. The gas thus produced becomes ionized, and out of this gas cloud the inner planets are formed in about the same way as are the outer planets.

It is difficult to say how the volatilization of the dust could be effected. It may have been mainly through heating by solar radiation — the same process that we observe in the formation of comet tails. If so it is difficult to understand why the gas should condense again, to form planets. However, there is the possibility that the condensation took place upon bodies considerably larger than the initial dust particles, but still small.

Another possibility is that the dust cloud had a rather high density so that mutual collisions between the particles were very frequent. At the high velocity which they must have in the region of the inner planets, such collisions will in general cause both the colliding particles to volatilize.

9. Are the inner planets and outer satellites formed at the same time?

If the meteoric gas cloud is volatilized and ionized more or less symmetrically around the sun (in the region of the inner planets) part of it must be expelled along the magnetic lines of force. According to the general laws of motion of ionized particles, a fraction will reach the region of the outer planets. As long as the gas is still ionized, it cannot approach the (outer) planets, because it is repelled by their magnetic fields. But when it has cooled down and become deionized, it is attracted by their gravitation. According to the rules that we have applied when accounting for the formation of the outer planets and the inner satellites, this gas must give rise to satellite systems of the outer planets. However, as the average atomic weight of the gas obtained by volatilizing the meteoric dust must be rather high, we must expect a high value of m'/V_{ion} and consequently a low value of M/r. This is just what characterizes the »outer satellite» group. Thus it seems possible that they have been formed at the same time as the inner planets.

In § 7 we have found that in order to account for the formation of the outer satellites, we must assume the arrival of a gas with $m'/V_{\text{ion}} = 3.4$. As we shall see later, (§ 14) this value must be increased by 50 % i. e. to $m' V_{\text{ion}} = 5.1$. If the meteoric dust had the same composition as the meteors which now reach the surface of the earth, the most abundant elements of it must be the following, the $m'/V_{\rm ion}$ values of which are given in the table,

Element	m'	$V_{\rm ion}$	$m'/V_{\rm ion}$
Fe	56	7.8	7.2
O	16	13.5	1.2
Ni	59	7.6	7.8
Si	28	8.1	3.5
Mg	24	7.6	3.2

Of course it is difficult to estimate their relative abundance in our meteoric dust, but a weighted mean of their $m'/V_{\rm ion}$-values might very well give that value ($= 5.1$) which is required to explain the position of the outer satellites.

10. Summary of B.

Our picture of the formation of the solar system is thus the following.

During its passage through space the sun, being initially a single star without planets, has passed a gas cloud and a dust cloud. As we know many clouds of both types, there is a large probability for both of these two occurrences.

When the sun passes the gas cloud, a part of the gas falls in towards the sun under the action of gravitation. Through the fall the gas is heated (gravitational energy being converted into heat), and when sufficiently heated it becomes ionized. As soon as it has been ionized the main force acting upon it is no longer the solar gravitation but electric and magnetic fields, especially the general magnetic field of the sun. Because of the laws of motion of charged particles in such fields, matter will accumulate in the equatorial plane and especially at that distance from the sun where the gravitational energy attains a certain critical value. This distance can very well be identified with the solar distance of the heavy planets.

Most of the gas is supposed to condense to planets. A small part, which is still uncondensed when the planets have been formed, falls towards the planets and the same process is repeated, giving rise to the »inner satellites» of Jupiter and Saturn, (including Saturn's rings) the gravitational energies of which have the same critical value as in the planetary system. Thus the outer planets and the inner satellites form the »first family».

The inner planets and the »outer satellite» systems of Saturn and Uranus cannot have been formed at the same time as the first family. Their formation could be explained through assuming that the sun also has passed through a dust cloud, which in some way has volatalized in the neighbourhood of the sun. The gas thus produced has been ionized, and through the same processes as those described above the inner planets and outer satellites have been formed as a »second family».

C. On the formation of planets and satellites.

11. The acceleration of the ions.

This and the following sections will be devoted to a closer study of the dynamics of the ions in the solar magnetic field.

Suppose that an atom of the primary cloud becomes ionized at the point (r_0, φ_0). According to § 2 it can only move along the magnetic line of force through this point, or more generally on the surface engendered through rotating this line of force around the sun's axis. The equation of the line of force is

$$r = R \cos^2 \varphi \tag{11.1}$$

where R is the distance from the dipole to the point where the line of force intersects the equatorial plane.

$$R = \frac{r_0}{\cos^2 \varphi_0}. \tag{11.2}$$

The drift of the ion (and electron) is composed of the motion along the magnetic line of force and the rotation around the axis of the sun. The latter is caused by the electric field from the polarisation of the sun (due to its rotation). If there is no considerable space charge in the environment, this field will immediately compel the ion (and electron) to take part in the solar rotation. However, if a large number of atoms, being initially at rest, are ionized at the same time it is evident that together they have a mass too large to be accelerated instantly. To express it in other terms, the ionized atoms form a cloud which has a finite electrical conductivity. Because of this the electric field decreases. A current in the ion cloud is produced and the force due to the interaction between this current and the solar magnetic field accelerates the ion cloud. Compare ref. (2) and (4).

Thus if a part of the cloud is ionized at a certain instant, it will at the first moment remain at rest and then, more or less slowly, begin to rotate around the axis of the sun.

12. On the point of equilibrium on a magnetic line of force.

The motion of ions and electrons parallel to the magnetic field (»along the magnetic line of force») depends upon the forces acting in this direction. This problem has already been treated in connection with the solar corona (3). To the forces dealt with in that paper we must add the centrifugal force f_c due to the rotation of the ions around the axis of the sun. (This force can be neglected in the corona).

As in a certain volume there must always be almost the same number of ions as of electrons, it is convenient to calculate the motion of an ionized atom (= ion + electron). If an ionized atom (mass $= m$) at the point (r, φ) rotates around the axis of the sun with the angular velocity ω, it is subject to a centrifugal force

$$f_c = m\,\omega^2 r \cos\varphi. \qquad (12.1)$$

If α and β are the angles which the magnetic field makes with the vector radius and with the centrifugal force we have[1]

$$\cos\beta = \cos\alpha\cos\varphi + \sin\alpha\sin\varphi = \frac{3\sin\varphi\cos\varphi}{\sqrt{1+3\sin^2\varphi}} = \frac{3}{2}\cos\alpha\cos\varphi. \qquad (12.2)$$

Consequently, in the direction of the magnetic field the following forces act upon the ionized atom:

Centrifugal force:

$$\frac{3}{2} m r\,\omega^2 \cos^2\varphi\cos\alpha. \qquad (12.3)$$

Gravitation

$$-\frac{k M_\odot m}{r^2}\cos\alpha. \qquad (12.4)$$

Magnetic gradient[2]

$$2\,\frac{2 E \vartheta}{r}\cos\alpha. \qquad (12.5)$$

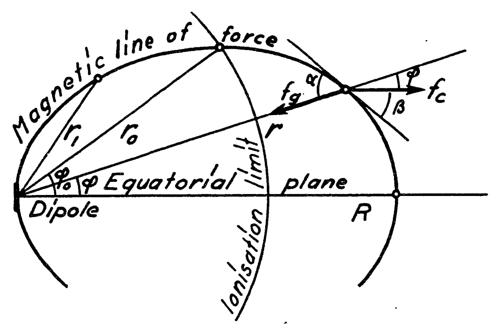

Fig. 3.

Pressure gradient

$$-\frac{1}{N}\left(\frac{\partial p}{\partial r}\cos\alpha + \frac{\partial \varphi}{r\,\partial\varphi}\sin\alpha\right). \qquad (12.6)$$

As in the paper on the corona (3) N denotes the total number of ionized atoms per cubic centimeter, p the gas pressure, and $2E$ the energy of each ionized atom. Further we have

$$\vartheta = 1 + \frac{\cos^2\varphi}{2(1+3\sin^2\varphi)}. \qquad (12.7)$$

[1] Compare ref. 2 § 8.

[2] See ref. 2, p. 12; and ref. 3, p. 12 and 14.

On their motion along the magnetic line of force the ionized atoms will find an equilibrium in the point (r, φ) where the sum $F_{\|}$ of these four forces vanishes. Since at this point the pressure will be a maximum, the pressure gradient can be neglected. Thus we obtain as the *condition for equilibrium*

$$\frac{3}{2} m r \omega^2 \cos^2 \varphi - \frac{k M_{\odot} m}{r^2} + 2 \frac{2 E \vartheta}{r} = 0 \tag{12.8}$$

or as $\cos^2 \varphi = \frac{r}{R}$

$$\frac{3 m r^2 \omega^2}{2 R} - \frac{k M_{\odot} m}{r^2} + 2 \frac{2 E \vartheta}{r} = 0.$$

Immediately after the ionisation has taken place, ω is zero. The point of equilibrium is then given by

$$r_1 = \frac{1}{2} \cdot \frac{k M_{\odot} m}{2 E \vartheta}. \tag{12.9}$$

The energy $2E$ of the ionized atom equals the gravitational energy minus the ionisation energy, but according to our assumptions (§ 3) the latter equals the gravitational energy at the point $(r_0 \varphi_0)$, where the atom has been ionized. Thus we have

$$2 E = \frac{k M_{\odot} m}{r_1} - e V_{\text{ion}} = k M_{\odot} m \left(\frac{1}{r_1} - \frac{1}{r_0}\right). \tag{12.10}$$

From (12.9) and (12.10) we obtain

$$r_1 = r_0 \left(1 - \frac{1}{2 \vartheta}\right) = r_0 \frac{2 (1 + \sin^2 \varphi_1)}{3 + 5 \sin^2 \varphi_1}. \tag{12.11}$$

Thus $\frac{r_1}{r_0}$ varies between 2/3 and 1/2.

Consequently, immediately after being ionized at the point $(r_0 \varphi_0)$ the matter will move along the magnetic line of force through this point to the equilibrium given by (12.11). When later it begins to rotate around the axis of the sun, the centrifugal force comes into action, so that the equilibrium is displaced outwards. At the same time the energy E decreases, because the particles move outwards. Further the whole gas is likely to cool down through radiation, etc. On the other hand there is a heating by the electric current producing the acceleration.

The action of the electric and magnetic field of the sun continuously increases the angular momentum

$$C = r^2 \omega \cos \varphi \tag{12.12}$$

of the atoms. When C has reached a certain value the vector radius is [according to (12.8)]

$$r = \frac{3}{2} \frac{C^2}{k M_\odot (1 - \Delta)} \tag{12. 13}$$

where

$$\Delta = \frac{4 E \vartheta r}{k M_\odot m}. \tag{12. 14}$$

At the point $r = r_i$ (given by 12. 11) we have $\Delta = 1$ and $C = 0$. When the particles move outwards, their temperature E, and consequently Δ, decrease.

Thus, a particle which is ionized at r_0 moves at first instantly inwards to r_1. Later its vector radius increases slowly when the rotation begins, and finally it reaches the equatorial plane at the distance R. A further increase in C cannot increase the vector radius.

13. On the recombination and condensation.

The question now arises how planets and satellites are formed out of the ionized cloud. According to LINDBLAD (5) there are good reasons to suppose that dust particles in interstellar space are formed through a process of condensation which he has studied in some detail. This process is a result of the great difference between the temperature ($\sim 10\,000°$) of interstellar gas and the temperature ($\sim 3°$) of the solid particles. He has also shown that in the solar system the same process can give rise to large bodies, for example »planetesimals» which later coalesce into planets and satellites.

It is reasonable to suppose that this process could be in action in our ionized cloud. The temperature corresponding to 12 *e*-volts (= the energy of the gas when it reaches the »ionisation limit») is about $100\,000°$, so that even if it has cooled down considerably before the commencement of condensation it is still of the same order of magnitude as in interstellar space. On the other hand the temperature of the solid particles would certainly be much higher than $3°$.

It is probable, though not quite certain, that the Lindblad process could start from a cloud which is ionized. If so, »planetesimals» could be formed directly out of our ionized cloud. On the other hand if it is necessary to have atoms as raw material we must assume that the ionized atoms recombine at first and later condense to planetesimals. Fortunately it does not matter very much for the purpose of our discussion which of these processes really occurs.

Let us suppose that a number of ions and electrons recombine when they are in equilibrium at the point (r', φ') and drift with the velocity $v' = \omega' r' \cos \varphi'$. The neutral particles (atoms or planetesimals) thus formed are not affected by the magnetic field and consequently they will move in an ellipse according to Kepler's laws. The angular momentum C of the motion is

$$C = r' \cdot v = \omega r'^2 \cos \varphi' = \sqrt{\frac{2}{3} k M_\odot r' (1 - \varDelta)} \tag{13.1}$$

according to (12.13). Introducing polar coordinates (r, θ) in the plane of the ellipse, its equation is

$$r = \frac{p}{1 - e \cos \theta} \tag{13.2}$$

with

$$p = \frac{C^2}{k M_\odot} = \frac{2}{3} r' (1 - \varDelta) \tag{13.3}$$

and the excentricity

$$e = \frac{1}{3}(1 + 2\varDelta). \tag{13.4}$$

Sooner or later the acceleration process will bring all ions to the equatorial plane, where, consequently, the density will be especially high, which condition favours the recombination of ions and electrons. Further the recombination is favoured by a low temperature. Thus it is of importance to study the *special case* when the recombination takes place in the equatorial plane (at the distance R) and $\varDelta$ is negligible. This gives

$$p = \frac{2}{3} R \tag{13.5}$$

$$e = \frac{1}{3}. \tag{13.6}$$

The neutralized particles will move in ellipses with excentricity $e = \frac{1}{3}$.

14. The two-third law.

If recombination takes place symmetrically along a ring R in the equatorial plane, the neutral particles will collide mutually, when moving in their ellipses. This smoothes out the excentricity so that the ellipses are transformed into a circle, having the same angular momentum as the ellipses. Its radius is

$$r_C = \frac{2}{3} R. \tag{14.1}$$

We have now arrived at what seems to be a very importantant law:

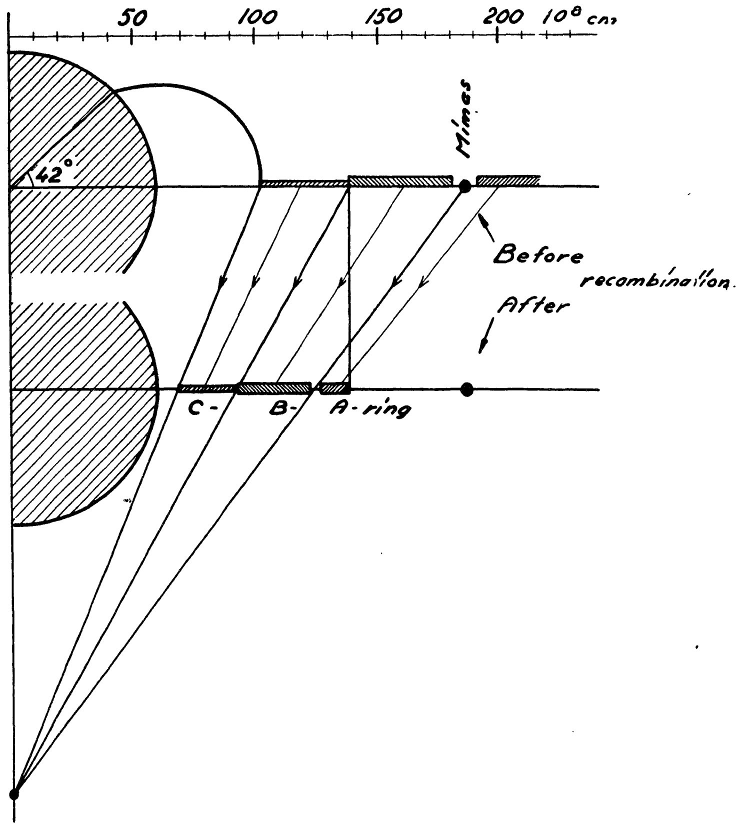

Fig. 4. Formation of the Saturnian ring system.

If our ionized gas recombines in or near the equatorial plane and the temperature of the gas is relatively low *the final orbit of the matter will be a circle with a radius equal to two thirds of the distance at which the recombination took place.*

This means that *the matter of an existing planet or satellite has initially been at a distance equal to 1.5 of its present distance from the central body.*

15. Application to the Saturnian system.

Applying this result to the Saturnian rings we must expect the matter constituting the rings to have been 1.5 times farther away from the planet before the recombination. *Enlarging the ring system 1.5 times* we find that *Cassini's division* (the dark space between outer and inner rings) *coincides with the (present) orbit of Mimas*, the innermost satellite. Further, *the beginning of the gauze ring coincides with the present outer edge of the outermost ring.* (See fig. 4.)

This indicates that the ring system may have been formed in somewhat the following way.

When the ionized matter around Saturn is accelerated, it is pressed down towards the equatorial plane, where recombination takes place. The neutral matter thus formed falls down to 2/3 of its former distance and moves, because of the symmetry, in circular orbits. If above ROCHE's limit it coalesces into satellites (through some unspecified process), if below ROCHE's limit it cannot form such large bodies but only small dust particles.

Suppose now that the condensation has started from outside and that all the (inner) satellites have been formed already. This has consumed most of the matter in the outer regions, but closer to the planet ionized matter still remains extending from the vicinity of Saturn's surface outwards to beyond the orbit of Mimas. However, all matter close to the orbit of Mimas is swept away by this satellite. Consequently, when after recombination the matter falls down to 2/3 of its former distance, we must find a dark spacing at 2/3 Mimas orbital radius. This »shadow» of Mimas can be identified with CASSINI's division. As all the matter of the outer ring (outside this division) has passed Mimas' orbit[1], part of it has been captured by Mimas. This explains why the outer ring is fainter than the inner ring.

When the outer parts of the rings have been formed, ionized matter in the vicinity condenses upon their particles. In other words, the small particles in the rings sweep the ions in exactly the same way as Mimas did, but perhaps not so completely. Thus we must find a »shadow» of the ring itself beginning at 2/3 of the distance to the outer edge. Just at that distance the intensity of the ring drops very much: the inner bright ring goes over into the dark gauze ring.

In order to account theoretically for *all* the data of the ring system we must also explain the situations of the outer edge of the outer bright ring and the inner edge of the gauze ring.

As has been pointed out by several authors, the outer edge of the ring system is probably given by ROCHE's limit. If dust particles outside this limit have a tendency to coalesce into larger bodies, the ring must become invisible at this limit. The mass of the ring is very small, and if concentrated to one or a few larger bodies these are not likely to be visible.

An inner limit to the ring system is given by the condition that the ions must not have hit the surface of the planet. If an atom is ionized at the distance r_0 it goes down immediately to the distance r_1 (see § 12), which must not be smaller than Saturn's

[1] The ellipses in which the particles move, cut the satellite orbit until their eccentricity has sufficiently decreased.

radius. The most favourable case is when the ionisation takes place in the equatorial plane ($\varphi_0 = 0$). If so we find from (12.11) and (12.7) with the help of (11.1): $r_1 = r_0(1 - \sqrt{0.2}) = 0.553\, r_0$; $\varphi_1 = \arcsin \sqrt[4]{0.2} = 42°$.

The present place of this matter must be

$$r = \frac{2}{3} r_0 = \frac{2}{3 \cdot 0.553} r_1 = 1.205\, r_1.$$

Consequently *the ring system cannot possible reach farther than to 1.21 times Saturn's radius.* According to observations the inner limit of the gauze ringe is 1.23 times Saturn's mean radius (1).

A survey of the theoretical and observational values (1) is given in Table 1 a and b. The agreement is rather satisfactory.

Table 1 a.

	Theory
$\dfrac{\text{Mean distance of Mimas}}{\text{Outer limit of Cassini's division}} = \dfrac{186}{122} = 1.52$	$\dfrac{3}{2} = 1.50$
$\dfrac{\text{Mean distance of Mimas}}{\text{Inner limit of Cassini's division}} = \dfrac{186}{117} = 1.59$	
$\dfrac{\text{Outer edge of } A\text{-ring}}{\text{Limit between } B\text{- and } C\text{-ring}} = \dfrac{138}{90} = 1.53$	$\dfrac{3}{2} = 1.50$
$\dfrac{\text{Inner edge of } C\text{-ring}}{\text{Mean radius of planet}} = \dfrac{71}{57.6} = 1.23$	$\dfrac{2}{3} \cdot \dfrac{1}{1 - \sqrt{1/5}} = 1.21$

Table 1 b.

	Identification	Theory 10^3 km	Observ. 10^3 km
Outer edge	Roche's limit	$2.44 \cdot 57.6 = 141$	138
Cassini's division	Shadow of Mimas	$\frac{2}{3} \cdot 186 = 124$	117—122
Beginning of dark ring	Shadow of outer edge	$\frac{2}{3} \cdot 138 = 92$	89—91
Inner edge	Shadow of planet	$\frac{2}{3} \frac{1}{1 - \sqrt{0.2}} \cdot 57.6 = 69$	71

(Mean radius of planet = 57.6; Mean distance of Mimas = $186 \cdot 10^3$ km.) Observational data compiled from R. D. S. Astronomy (1).

There is an alternative explanation of Cassini's division: that it is a resonance phenomenon. Particles revolving at that distance should be sorted out, because their period is exactly one half the period of Mimas. Unfortunately it is very difficult to judge from the dimensions which explanation is correct, because if the ratio of the periods is 0.5 the ratio of the orbital radii is $0.5^{2/3} = 0.63$, which is very close to our value $2/3 = 0.67$, which must be corrected somewhat if $\Delta \neq 0$.

There is no doubt that such a resonance effect exists. Among the asteroids there are very conspicuous gaps corresponding to 1/2, 1/3, 2/5 etc. Jupiter's period.

On the Saturnian rings there are also several dark markings at distances where the periods are commensurable with one of the satellite periods. These markings, however, are very faint, and not at all comparable with the broad Cassini division. Certainly we should expect the resonance with half the period of Mimas to be more pronounced than the other, but the observed difference seems to be much too large to be explained in this way only.

That the drop in intensity (at the border between bright ring and gauze ring) occurs at 2/3 the outer edge can certainly not be explained as a resonance phenomenon.

Against our interpretation it may be objected that, when the ionized atoms recombine, they will not at once go down to a circle at 2/3 the distance but move in excentric ellipses for some time until their paths are smoothed out by mutual collisions. If so they must collide also with the matter which has condensed already so that no structure with sharp shadows can be produced. However this argument is valid only for the particles recombining exactly in the equatorial plane. The ellipse of a particle which has started from a point at some small distance from the equatorial plane lies in a plane which makes an angle with the equatorial plane, and the ellipse intersects this plane in two points at the distance 2/3 r. Consequently, it does not interfere with the matter which has condensed already if this forms a flat ring in the equatorial plane. According to a theorem by LINDBLAD (M. N. 94 p. 231, 1934) the flatness can be interpreted as a result of slight inelastic collisions between the particles.

§ 16. **Application to the asteroids.**

Fig. 5 shows the number of asteroids at different distances from the sun. The graph is plotted from data of 1512 asteroids (6). The main body of them are situated inside Jupiter's »shadow», i. e. inside 2/3 Jupiter's distance, but outside the »shadow» of their own outer edge, or outside 2/3 of their further limit. Thus the main asteroid ring is very similar to the Saturnian *B*-ring.

The ratio between the masses of Jupiter and of the sun is 10^{-3}, whereas the ratio Mimas-Saturn is about $0.6 \cdot 10^{-7}$. Consequently, we must expect Jupiter to disturb the asteroids much more than Mimas disturbs the Saturnian rings. One of the consequences is that the resonances at commensurable periods give very conspicuous gaps in the asteroid plot, whereas they give only faint markings on the rings.

Another result is that not very much is left of the matter originating from outside Jupiter's orbit. Most of this has very probably been captured by Jupiter. Only a few asteroids are found outside 2/3 of Jupiters orbit. Thus there is no real correspondence to the outer Saturnian ring. As mentioned in § 15 this ring is fainter than the second ring, probably because even the small Mimas has been able to capture a considerable part of the matter passing its orbit. Then it is very reasonable that the much larger Jupiter could capture practically all of it.

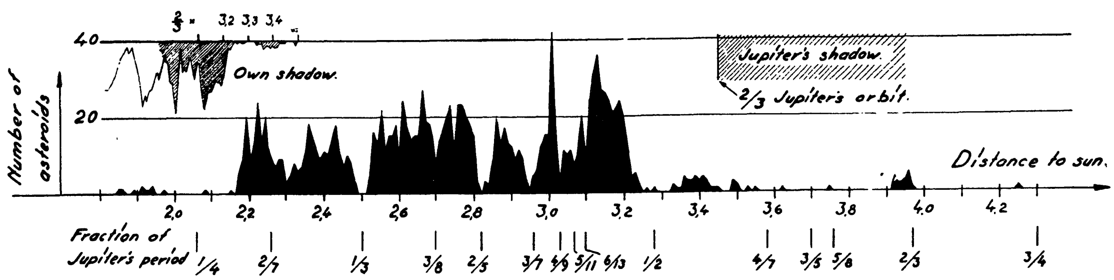

Fig. 5. The asteroids.

Fig. 5 shows that immediately inside Jupiter's shadow (2/3 Jupiters orbit) the number of asteroids begins to increase. The increase is interrupted by a resonance gap (period = 1/2 Jupiter's period), but after that the main part of the asteroids follow. Their own shadow (2/3 their present distances[1]) is marked in the figure. The inner limit of the asteroids agrees rather well with the beginning of their own shadow.

It is remarkable, although perhaps fortuitous, that almost exactly where the shadow of the asteroids ends, we find the planet Mars. In fact the ratio between the orbit of Mars and the innermost edge of the asteroid ring is 1.52 : 2.18 = 0.70 (instead of 2/3 = 0.67).

§ 17. A relation between mass and orbital radius.

The structure of the Saturnian and asteroid rings has given us some valuable hints concerning the very difficult question of how the condensation takes place. In both cases the formation of a satellite or planet appears to have been stopped at a half-way

[1] This is exact only if their orbits are circular. However it is impossible to take account of the excentricity because it is subject to secular variations, so that we do not know its value when the planets were formed.

stage: The recombination of the ions is finished and the neutral matter thus produced has agglomerated to dust particles or asteroids, but the coalescence to larger bodies, which probably would have been next process, has been prevented. The cause of this is obvious for the Saturnian rings: they are situated below the Roche limit. It is more difficult to understand why the asteroids have not coalesced into a larger planet. It may have some connection with the very low density of the original ion cloud in this part of space. (In reality it is perhaps more difficult to understand why at all other places in the planetary system large bodies are formed instead of rings of small bodies! As this is the rule, however, it is reasonable to suppose that some combination of mechanical and electric forces can affect it.[1])

Following upon the above, let us suppose that the first step in the formation of a planet (or satellite) is the formation of a ring like the asteroid ring or one of the Saturnian rings. If one planet has been formed already, the ring will be situated a certain fraction of its orbital radius from the central body. If such a ring coalesces into a new planet, its situation in relation to the first planet depends upon the density function $\vartheta(r)$ of the matter constituting the ring. If the density increases with r (distance to the central body) the mean angular momentum of the ring, and consequently the angular momentum of the planet, must be larger than if the density decreases when r increases. Thus, the relative distance between the first and the second planet must be the smaller the larger $\frac{d\vartheta}{dr}$ is.

From the present masses of the planets we can estimate the course of $\vartheta(r)$, because the larger the mass the larger the initial density. Thus taking $Q = \frac{m_{n+1}}{m_n}$ (m_n and m_{n+1} being the masses of two adjacent planets) as a rough measure of $\frac{d\vartheta}{dr}$, we should expect that Q would be a uniformly decreasing function of $q = \frac{r_{n+1}}{r_n}$ (r_n and r_{n+1} being the orbital radii of two adjacent planets).

Fig. 6 is a semilogarithmic Q-q-plot. If the masses are unknown, as is the case with the Uranian and some of the outer Saturnian satellites, Q is put equal to D^3_{n+1}/D^3_n (D = diameter).

At the first inspektion the points seem to be irregularly distributed. However, if we take out the points belonging to the »first family» (outer planets and inner satellites) all these are situated rather close to a straight line. The Q-values decrease

[1] MAXWELL's criterion of stability for a ring is not applicable because his treatise takes no account of electromagnetic forces.

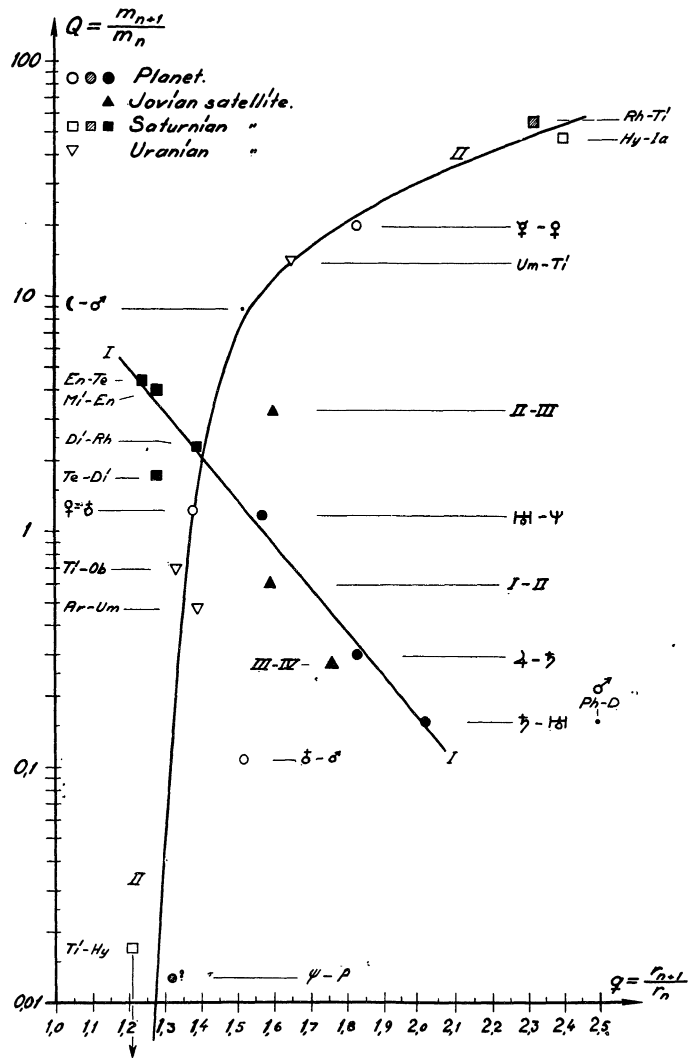

Fig. 6. Connection between masses m_n and orbital radii r_n of planets and satellites.

when q increases. This is qualitatively what we had expected. Thus there is some evidence that for these bodies the condensation process is of about the kind indicated above.

In the q-Q-diagram all other points except one (Earth-Mars) are situated so that a smooth curve can be drawn through them. This curve has quite a different character from that of the first curve, indicating that also another type of condensation process has been in action. In a linear diagram all dots (except Earth-Mars) lie very close to a straight line.

It is not easy to understand why the dots should lie on two different curves. The fact that they do so, however, strongly confirms our result that the planetary system has been formed through two different processes (gas cloud and dust cloud invasion).

It is remarkable that the dot of Titan-Rhea lies close to the second curve. According to our identification Rhea belongs to the inner and Titan to the outer satellites.

Pluto-Neptune also falls on the second curve. These planets have the same gravitational energy as the outer satellites (see Fig. 2). Further the density of Pluto is probably very high (about 4) which also indicates that it may belong to the »second family». If the meteoric dust cloud had already partly volatalized very far away from the sun, the invasion of the dust must have been accompanied by a heavy gas of the same kind as that forming the outer satellites. This would give planets in the same energy region as that of the outer satellites. Pluto may very well have been formed in this way.

As the Earth-Mars dot is the only one which does not fall on either of the curves, one must seek the cause of this exception. We should expect it to fall on the second curve because the Earth and Mars are both inner planets. If we suppose that it does so, and further assume that there is no error either in the q-value, or in the mass of Mars, we can calculate the mass which the Earth ought to have. *The mass we find agrees with the mass of our Moon!*

What conclusions might be drawn from this is difficult to say. The Moon's large mass and large distance from the Earth have already shown that it is no ordinary satellite. Instead, the Earth and the Moon form a sort of double planet. The planetary character of the Moon is stressed by the fact that the Mars-Moon-dot falls on the Q-q-curve. The densities of Mars and the Moon are almost equal (3.33 and 3.96).

The extremely small fifth Jovian satellite gives a dot far outside the diagram. The abnormal satellites of Jupiter and Saturn are probably captured asteroids and should of course not be represented; nor should the ratio Jupiter-Mars, because the asteroids are situated between these planets.

The Martian satellites give a dot not very far from the first curve. (Their gravitational energy is only about 1/30 of that normal for »the first family».)

The fact that practically all the Q-q-dots fall close to one of the two curves indicates that the formation of the planets and satellites has been a process far more regular than has hitherto been supposed. The laws regulating this process, or rather the two different processes, are, however, still unknown. If the regularity of the diagram is caused by the process of formation, as appears likely, it seems that neither the orbital distances nor the masses can have changed very much since the time of the genesis (or have changed in a very regular way).

Summary of C.

The formation of planets and satellites out of the ion cloud is studied. When the invading gas cloud has been ionized it begins slowly to take part in the solar rotation. The equilibrium of the ions is calculated.

The condensation of the cloud to planets and satellites is supposed to take place according to LINDBLAD's condensation theory. It is shown that when ions and electrons of the cloud recombine to neutral atoms these fall down to $2/3$ of their original distance from the central body. This law explains the structure of the Saturnian and asteroid rings.

If two adjacent planets or satellites have orbital radii r_n and r_{n+1} and masses m_n and m_{n+1}, we should expect theoretically that there is a relation between $q = r_{n+1}/r_n$ and $Q = m_{n+1}/m_n$. This is confirmed, but the connection is not the same for the bodies formed out of the gas cloud as for those from the dust cloud.

References.

1. RUSSELL, DUGAN, STEWART, Astronomy, Boston 1926.
   All numerical values are taken from this book.
2. H. ALFVÉN, On the motion of a charged particle in a magnetic field; Ark. f. mat., astr. och fysik, *27 A*, N:o 22, 1940.
3. ——, On the solar corona, Ark. f. mat., astr. och fysik, *27 A*, N:o 25, 1941.
4. ——, Remark on the rotation of magnetized sphere with application to the solar rotation. Ark. f. mat., astr. och fysik, Bd 28 A, N:o 6, 1942.
5. B. LINDBLAD, A condensation theory of meteoric matter and its cosmological significance. Nature, 135, p. 133, 1935.
6. Kleine Planeten, Jahrg. 1941, Copernicus-Institut, Berlin.

Stockholm, Sept. 1941. K. Tekniska Högskolan.