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ON THE COSMOGONY OF THE SOLAR SYSTEM

III

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HANNES ALFVÉN

WITH 6 FIGURES IN THE TEXT

COMMUNICATED DECEMBER 5th 1945 BY BERTIL LINDBLAD AND K. G. MALMQUIST

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§ 1. In two previous papers¹ a theory of the formation of the solar system has been proposed. The theory can be applied to the planetary system as well as the satellite systems. Of its results we shall here especially discuss the classification of the planets and satellites into two groups, called the first and the second family. The theoretical treatment of the formation of the solar system showed that all bodies could not have been formed at the same time and in exactly the same way. A division into (at least) two groups was necessary. Empirical data confirmed the classification.

Any theory of the origin of the solar system necessarily includes many speculative elements. The empirical relations, however, which were found during the theoretical investigation, seem to be so important as a basis for every attempt to understand the formation of the solar system, that it is worth while to try to present them independently of any theory. In order to determine which are the empirical facts and where the theoretical speculations come in, we shall here (§§ 2—4) make the same classification as in II but without any reference to the theory.

A. Classification of planets and satellites.

\S 2. The q-Q diagram and the density.

In II the classification of the members of the solar system was based on the theory of the formation of the system. Here we shall make the classification in a purely empirical way and choose three different grounds for the classification of a planet or satellite:

¹ H. Alfvén, On the cosmogony of the solar system I, Stockholms Observatoriums annaler Bd 14, No 2, 1942; II, Ib. Bd 14, No 5, 1943. Referred to as I and II.

- I. The situation of its dots in the q-Q diagram.
- 2. The density.
- 3. The gravitational potential in relation to the central body around which it revolves.

In order to show the *abnormal* character of a few very small bodies, the eccentricity and inclination of their orbits are also taken into consideration.

The q-Q diagram is constructed in the following way. For each pair of consecutive members n and n+1, (counted from the central body around which they revolve) of the planetary or one of the satellite systems, the ratios

$$q = \frac{r_{n+1}}{r_n}$$

and

$$Q = \frac{m_{n+1}}{m_n}$$

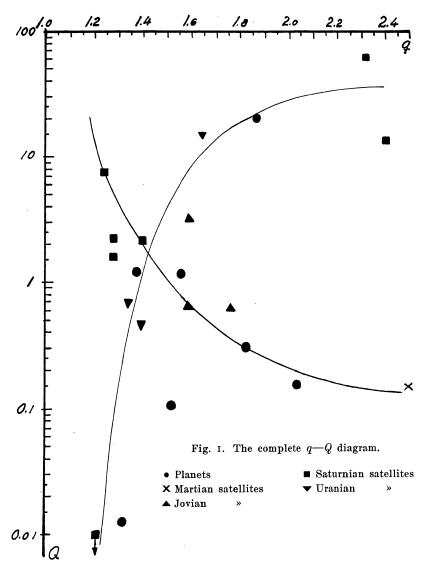
of their orbital radii r_{n+1} and r_n and of their masses m_{n+1} and m_n are calculated and plotted in a diagram (Fig. 1). If the masses are unknown (as for the Martian and Uranian satellites) the cubes of the diameters are used instead.

In the figure all pairs of planets are represented except the pair Mars—Jupiter, of which the dot falls far outside the upper right hand corner. Of the satellites, the typically »abnormal» cases (Jupiter VI—XI, Saturn IX) with orbits of large eccentricity (>0.15) and large plane inclinations (>25°) are excluded. All other satellites are represented except the pair consisting of the fifth and the first Jovian satellite, the dot of which falls far outside the upper right-hand corner.

The dots are on the whole situated close to one rising and one falling curve, which may indicate that there are two different families of bodies. As the curves intersect and the dots are distributed over a rather large area, no certain conclusions can be drawn from this diagram alone. It must be noted that all the bodies situated on the rising curve have a higher density than those on the falling curve.

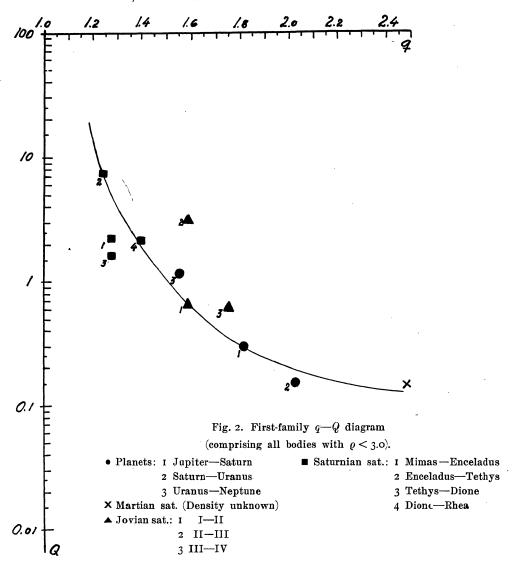
One of the most conspicuous differences between the bodies in the solar system is the difference in density. The inner planets, for example, have densities ranging from 3.8 to 5.52, whereas the giant planets have densities between 0.71 and 1.58. This has been assumed to be a consequence of the difference in mass alone (the bigger the planet, the lighter the substances it can retain). But if we examine the Saturnian satellite system

¹ As in II the data are taken from the tables in Russel, Dugan, Stewart, Astronomy I, Boston 1926; Russel, Solar System and its Origin, New-York 1935; W. de Sitter, M. N. 91, p. 734, 1931; G. Struwe, Veröff. Univ. Sternwarte Berlin-Babelsberg VI, Heft 4, 1930. Compare II, Table 1, p. 5.



we find the reverse conditions; the big Titan has a density of 3.24, whereas the small Mimas has a value lower than 0.54. Hence the difference in density is not likely to be a function of the mass of the body. A classification of the bodies in the solar system with regard to the density is very natural.

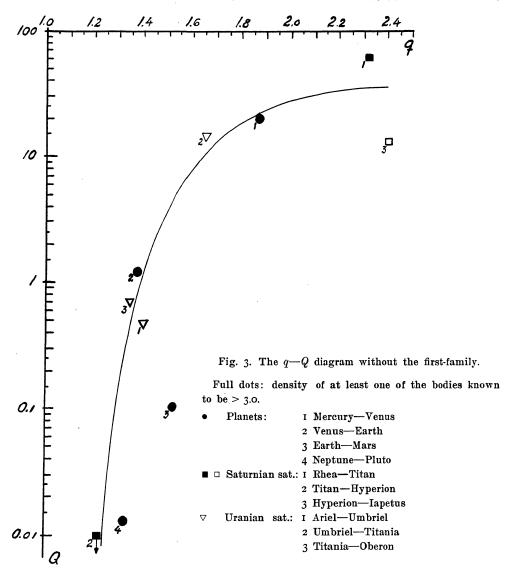
If we divide the bodies into one *family* with low density and one with high density, we must draw a limit between them. It is obvious that the giant planets and the Saturnian satellites Mimas (density $\varrho = 0.11-0.54$). Enceladus ($\varrho = 0.12-0.62$), Tethys (0.20-0.99), Dione (0.47-1.97), and Rhea (0.36-1.55) should belong to the first family. If we make a q-Q diagram for these bodies alone, all of which have densities below 1.58, we find that all their dots lie on the falling curve (Fig. 2). Close to this curve lie also the three dots representing the four big Jovian satellites ($\varrho = 2.66$, 2.86, 2.16, 1.32).



The body coming next with regard to density is Titan ($\varrho=3.24$). Both of its dots lie very far from the falling curve. Hence, if we draw the limit between the families at a density of about 3.0, all the members of the first family lie on the falling q-Q-curve. The q-Q diagram of the first family, defined as including all bodies with $\varrho<3.0$, is shown in Fig. 2. All dots lie close to a monotonic curve (perhaps with some doubt regarding the dot for the second-third Jovian satellite. This curve could very well be drawn through the dot of the Martian satellites. The density of these is not known.

Through all the remaining dots — except the Earth—Mars dot — a monotonic curve could be drawn (Fig. 3). Of the bodies represented by them we know the densities only

² Of all the bodies that we have counted among the first family, the second Jovian satellite has the highest density.



of the inner planets and Titan. For the Saturnian satellites Hyperion and Iapetus the densities are known whithin the limits 1.38—6.24 (Hyperion) and 1.03—4.43 (Iapetus). The densities of the Uranian satellites are unknown.

If the second family is to comprise all bodies with densities above 3.0, the inner planets, the moon, and Titan belong to this. The three dots representing the four inner planets lie in such a way that a curve through all of them would have a character quite different from the first-family-curve. If we draw a curve so that it passes close to the dots representing satellites which may belong to the second family, this curve goes through the Mercury—Venus and the Venus—Earth dots, but far away from the Earth—Mars dot. This dot must be regarded as an exception which should be discussed later.

The dots representing Neptune—Pluto and Rhea—Titan both lie at such places that the second-family curve could be drawn close to them. In both cases one of the members (Neptune and Rhea) belongs to the first family. Both Pluto and Titan have high densities (Pluto probably about 4), so these two dots consist of members of both families.

The Titan—Hyperion dot, through which the second-family curve can be drawn, may represent two second-family members, as even Hyperion may be of a high density (the value being between 1.38 and 6.24). The same is the case for the Hyperion—Iapetus dot (Iapetus has the density 1.03—4.43).

All the three dots representing the four Uranian satellites fall close to the curve drawn through the members of the second family, which we have already identified. One of the dots is remote from the first-family curve. Two lie not very distant from it, but they agree much better with the second-family curve. This indicates that all the Uranian satellites may belong to the second family. As their densities are unknown, this cannot be verified.

To sum up, we have found that the densities and the q-Q diagram indicate that the bodies of the solar system can be divided into two families:

The first family, characterized by a low density ($\varrho < 3.0$) and by the fact that the dots lie on the falling curve. This comprises the giant planets, the four big Jovian satellites and the inner Saturnian satellites (Mimas—Rhea). It is possible that the Martian satellites belong to this family.

The second family, characterized by a high density $(\varrho > 3.0)$ and by the fact that the q-Q dots lie on the rising curve. This comprises the inner planets, Pluto, the Saturnian satellite Titan, and probably also Hyperion and Iapetus. The Uranian satellites are likely to belong to this family.

§ 3. The gravitational potential.

That the division of the bodies into two families is not arbitrary but has a real significance, is shown if we consider their gravitational potentials $\frac{M}{r}$ (M = mass of the central body around which the body in question revolves at the distance r). We find that the gravitational potential is of the same order of magnitude for all the members of the first family. In the planetary system it lies within the limits $4.41 \cdot 10^{18}$ and $25.5 \cdot 10^{18}$ g cm⁻¹. In the Jovian system the limits are 10.1 and 45, in the Saturnian system 11.0 and 31.4. Consequently, all members are comprised between the limits $4.41 \cdot 10^{18}$ and $45 \cdot 10^{18}$.

¹ Tabulated in II Table 1, p. 5.

If we pass to the second family, we find that the inner planets have potentials between 87 and $342 \cdot 10^{18}$ i.e. well above the potentials in the first family. In the second family we must also count Pluto $\left(\frac{M}{r} = 3.09\right)$, Titan $\left(\frac{M}{r} = 4.77\right)$, and probably also Hyperion $\left(\frac{M}{r} = 3.92\right)$, Iapetus (1.63), and the Uranian satellites $\left(\frac{M}{r} = 1.62 - 4.95\right)$. All these bodies have gravitational potentials between 1.62 and 4.95 · 10¹⁸ g cm⁻¹. These values are below the first-family values, but for a small overlapping (the values of Titan and Ariel being just a little above that of Neptune). Thus, the members of the second family have gravitational potentials above as well as below the energies in the first family. 2

The two families are characterized not only by differences in density and different positions in the q-Q diagram but also by different gravitational potentials.

Some of the classifications may be somewhat dubious because we do not know the densities of all bodies, because the two q-Q curves intersect so that it is not possible to decide to which curve the dots near the intersection are to be counted, or because the energy ranges overlap somewhat. There is, however, one fact only which is in definite disagreement with the general rules we have found, namely the situation of the Earth—Mars dot. We should expect it to fall on the rising q-Q-curve. In order to place it there with an unchanged q-value, we must increase the Q-value by a factor 100. If we use the mass of the Moon instead of that of the Earth we obtain a dot in the right position. This would mean that in this respect the Moon is to be counted as a planet. As has often been pointed out, the Moon is no ordinary satellite. Its mass in relation to the Earth is much larger than that of any other satellite in relation to its primary. The Earth—Moon system is more like a double planet. The facts that the Earth—Mars dot does not obey the rules to which we know no other exception and that the Moon-Mars dot fits well into the curve, indicate that the Moon forms together with Mars a special group of the second family. As the Mercury—Venus and the Venus—Earth dots are quite normal, Mercury, Venus, and the Earth must form another group.

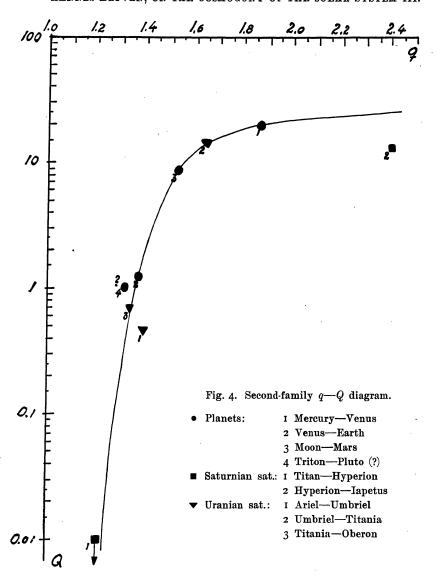
It has often been pointed out that Neptune and Triton form a system somewhat similar to the Earth—Moon system. It is possible that Triton and Pluto form a group

¹ The gravitational potentials of bodies in the planetary system seem in general to be somewhat smaller than the values of corresponding bodies in the satellite systems. Compare I p. 13-14.

² For the theoretical explanation of this, see I §§ 7-9.

³ The Q-value of Hyperion-Iapetus is lower than that of Mercury-Venus, although the latter q-value is less. The mass of Hyperion is known only very approximately, so there is no definite discrepancy present.

⁴ An independent argument for counting the Moon and Mars in a separate group is given in II p. 24—28. It is based upon considerations of the density as a function of the orbital radius.



together, analogous to the Moon—Mars group. We know too little about Triton to decide this with certainty. When counted as a planet, its gravitational potential (in relation to the Sun) would fall near the upper limit of the second-family range and the q-Q-dot for Triton—Pluto would be situated not far from the rising curve.

The revised second-family diagram (with the Moon and Triton (?) counted as planets) is shown in Fig. 4.

We have studied the q-Q diagram for pairs of bodies belonging to the same family. In order to complete our analysis we shall also study how those dots are situated which represent adjacent bodies of different families. There are four such cases: Mars—Jupiter, Neptune—Pluto (dubious if Triton is considered as forming a group together with Pluto),

the Jovian satellites V—I, and Rhea—Titan. Of these four cases the Rhea—Titan and the Neptune—Pluto dots lie not too far from the second-family curve. The Jovian satellites V and I give a q-value 2.33 and a Q-value (from the cubes of their diameters, since the mass of V is unknown) of about 100,000. The corresponding dot is very far from both curves. Mars—Jupiter give q=3.43 and Q=2,900. Both the q- and Q-value are much above the values of the other dots. The second-family curve could be drawn through this point, but one does not have the impression from the diagram that it will turn this way. Consequently, of the four "mixed" dots, two are not very far from the second-family curve, one is very distant and one probably rather distant from the curves. They thus appear to obey no simple rule.

§ 4. Result of the classification.

As pointed out above, the second family seems to consist of three groups: 1. The low-potential group containing Pluto and possibly Triton, the Saturnian satellites Titan—

Table 1.

Body	Density ϱ	q-Q	Potential $\frac{M}{r}$
Mercury	II	l II	II
Venus	II	II	II
Earth	II	II?	II
Moon	II	II?	II?
Mars	II	II?	II.
Jupiter	Ī	ī	I
Saturn	ī	I	I
Uranus	ī	I	T
Neptune	r	1?	1?
Pluto	II?	II??	II .
Martian sat.	?	1?	II??
Jovian sat. V	?	?	II
» I—IV	T	I	I
Mimas-Rhea	I	I(?)	ı
Titan	II	TT	II?
Hyperion, Iapetus	?	II	II.
Uranian sat	· ?	II(?)	II
Triton	?	II(!)	II ??
TIIUUII	ţ.	TILL	17.1.

Definition: Family I: $\varrho < 3.0$; q-Q-dot on the falling curve; $4.5 \cdot 10^{18} < \frac{M}{r} < 50 \cdot 10^{18}$ Family II: $\varrho > 3.0$; q-Q-dot on the rising curve; $\frac{M}{r} < 5.0 \cdot 10^{18}$ or $> 50 \cdot 10^{18}$ Compare II, p. 5, Table I

Iapetus, and the Uranian satellites. 2. The Mercury—Venus—Earth group. 3. The Moon—Mars group (to which the fifth Jovian satellite perhaps also belongs). There is no reason for a division of the first family into groups (at least not as long as the classification of the Martian satellites is not definite).

The three different grounds for the division of the bodies into two families are summarized in Table 1. A question mark alone signifies that no classification is possible. A question mark after a figure means that the classification is not absolutely unarbitrary and two question marks that it is uncertain.

B. Extrapolation to unknown bodies.

§ 5. In most attempts to explain the distribution of planets and satellites, it has been necessary to introduce "missing" planets or satellites. In our case this is not necessary. The q-Q diagram does not compel us to assume the existence of unknown bodies. On the other hand it allows us to do so. The different groups may very well be continued outwards or inwards by members still undiscovered. We shall call such members ultraplanets or infraplanets (ultra- or infra-satellites).

Since the q-Q-diagrams, as presented in Figs. 2 and 4, show rather well defined curves from which there is no exception among the bodies we know (after the revision of the Earth—Mars dot), we may have some confidence in using them for extrapolation to unknown bodies. If an unknown member A of a group is situated, for example, beyond the outermost known member B of the group, we might expect that the q-Q-dot for A—B would fall on the curve representing the family in question. This gives a relation between the mass and the orbital radius of A, if the same quantities are known for B.

In this way it is easy to predict the properties of a planet situated outside Pluto or inside Mercury. We shall not enter into a detailed discussion of this but, with regard to what follows, confine ourselves to investigating what planets are to be expected in the range of the asteroids. Between Jupiter and Mars there could be a first-family infraplanet, Infra-Jupiter, as an extrapolation of the group of the giant planets, and a second-family ultraplanet, Ultramars, as an extrapolation of the Moon—Mars group. There is also a possibility that the Mercury—Earth group could have an ultraplanet, Ultratellus.

If there is a planet inside Jupiter belonging to the Neptune—Jupiter group, it must have a very small mass, since we know no large first-family planet inside Jupiter. Thus, in order to find the orbital radius of the hypothetical Infrajupiter, we must follow the first-family curve to very large Q-values. The largest observational Q-value within the first family is Q = 7.57 (Eneceladus—Tethys). The corresponding q-value is

$$q = 1.24.$$

This value ought to be an upper limit for the q-value when Q tends to infinity. An extrapolation must, of course, be uncertain, but a q-value of about 1.2 seems likely. This corresponds to a solar distance of $\frac{5 \cdot 2}{1 \cdot 2} = 4.3$ times the Earth's distance.

In order to find a corresponding very small member of the second family, we have to extrapolate the second-family q-Q-curve to Q=0. Proceeding as above, we find from II (Table 1) that the smallest observational Q-value is Q=0.0008 (Titan—Hyperion) and this corresponds to

$$q = 1.21.^2$$

Thus, the limiting value is likely to be about the same as for the first-family infraplanets, or about 1.2.

Consequently, we might expect a planet *Ultramars* at a distance of 1.20·1.52=1.82 and perhaps also a planet *Ultratellus* at 1.20·1.00== 1.20.

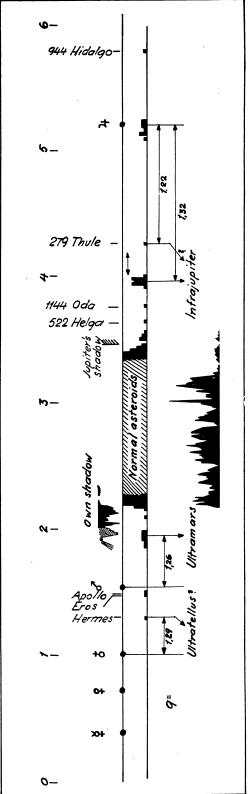
C. Classification of the asteroids.

§ 6. The asteroid ring.

According to the periodical »Kleine Planeten» (1942) the number of discovered asteroids amounts to 1542. Most of these have mean solar distances between 2.2 and 3.4 (Earth = 1). As has often been pointed out, they form a »ring» similar to the Saturnian ring system.

¹ The two semi-theoretical formulae (6.3) and (6.7) in II give for $Q=\infty$ the values q=1.22 and q=1.29.

² The extrapolation of the semi-theoretical formula in (3.7) with $\alpha=1.75$ gives, for Q=0, q=1.17.



Below the main diagram: normal asteroids in diminished scale. The asteroids and adjacent planets. ō. Fig.

The gravitational potential $\frac{M}{r}$ of the asteroid ring is the same as that of the Saturnian rings. In fact the limits 3.4 and 2.2 to the asteroid ring correspond to 10^{-18} $\frac{M}{r} = 39$ and 60 g cm^{-1} , whereas in the Saturnian system the potential limits are: for the A-ring 40-46, the B-ring 48-63 and the C-ring 63-80.

We have hitherto made no use of the theory expounded in I and II, but have carried out the classification of the planets and satellites on purely empirical grounds.

In the classification of the asteroids, however, it is useful, if not essential, to introduce some of the theoretical results of II. According to what was called "the two-thirds law" a body with the orbital radius r_0 , gives a "shadow" at $\frac{2}{3} r_0$, so that there could be no body at this place. Applied to the Saturnian rings this law explains the Cassini division as the "shadow" of Mimas, which has an orbital radius of $\frac{3}{2}$ times that of the Cassini division. Further, the rapid drop in intensity which marks the border between the B- and the C-ring is situated at $\frac{2}{3}$ of the outer edge of the A-ring. This was explained as the beginning of the shadow of this ring. (The more complicated explanation of the inner edge of the C-ring may not be correct. In the theory an expression for a "magnetic force" was introduced, which was taken from a paper on the solar corona (Ark. f. Mat. astr. och fysik Bd 27 A N:o 25 1941) where it was introduced in an erroneous manner).

If we wish to apply the two-thirds law to the asteroids, it is of importance to calculate where Jupiter's »shadow» falls. We find that this is at the distance $^2/_3 \cdot 5.20 = 3.46$. This is almost exactly where the main group of asteroids begin (Fig. 6). Only seven »stragglers» are found just a little above this value, in fact up to 3.55.

From Jupiter's shadow the asteroids fill out the range down 2.17. There are, however, a series of gaps, or at least points, where the number of asteroids is low. As is well known, this is due to resonance effects depending upon the fact that the time of revolution of an asteroid there would be commensurable with that of Jupiter. The most conspicuous gap is situated at 3.27, corresponding to a period equal to half of that of Jupiter. The number of asteroids above this gap is rather small, in fact only 39.

According to the two-thirds law no asteroids are to be expected below ²/₃ of the solar distance where they begin. The »shadow» of the asteroids themselves is shown in

¹ According to the theory, this agreement is mostly fortuitous. The agreement in energy between the giant planets and the inner Saturnian satellites has a physical significance. The Saturnian ring system is connected to the Saturnian satellite system and the asteroids have a certain connection to Jupiter, which results in an indirect connection between the two ring systems.

the upper part of the figure, where the distribution plot is shown diminished to $^2/s$ (and inverted). The 39 asteroids above the big resonance gap seem to be too few to give an appreciable shadow, but as soon as the shadow of the asteroids below the resonance gap occurs, the asteroid ring ends. In fact, the shadow of the resonance gap is situated at $^2/_3 \cdot 3.27 = 2.18$, and but for a single straggler the asteroids end at 2.15.

Consequently, there is reason to consider all asteroids between 2.15 and 3.55 as "normal asteroids", forming a ring analogous to the Saturnian B-ring. Of the asteroids outside these limits there is a group called "the Trojans", which have the same distance as Jupiter (5.09 < a < 5.24 against a = 5.20 for Jupiter). Their formation is probably closely connected with that of Jupiter. They may perhaps be considered as small parts of the matter which was accumulated at the distance of Jupiter. For some reason, small fractions of this were not captured by Jupiter, and these remnants have later formed the Trojans. The "resonance" with Jupiter, due to the equality of the periods, has of course, greatly affected their situation.

For the twelve Trojans (588, 617, 624, 659, 884, 911, 1143, 1172, 1173, 1208, 1404, and 1437) the inclination of the orbital plane varies between $i = 3.1^{\circ}$ and $i = 33.7^{\circ}$. Their angles of eccentricity are up to $\varphi = 8.6^{\circ}$.

It is not unlikely that also other planets have small components which, like the Trojans, have about the same solar distance. Indeed, we find two asteroids at the same distance as Mars (although they do not have the special type of motion as the Trojans). For one of them, the renowned Eros, the distance is a = 1.46, the other, the extremely small Apollo, has a = 1.48, both values very close to a = 1.52 for Mars. Compared with the Trojans their inclinations are about the same ($i = 10.8^{\circ}$ for Eros and $i = 6.4^{\circ}$ for Apollo). The eccentricity of Eros is only a little higher ($\varphi = 12.9^{\circ}$) than that of the Trojans, but Apollo is very eccentric ($\varphi = 34^{\circ}$).

Besides the asteroids already discussed, there are two large groups at about 1.9 and 3.95, and also five single specimens.

We have found in § 5 that in the region of the asteroids three infra- or ultraplanets might be expected, Infrajupiter, Ultramars, and Ultratellus.

To start with Ultramars, this should occur at about 1.2 times the distance of Mars. One of the two groups of asteroids is in about the right position. This group contains 11 members (434, 1019, 1025, 1103, 1139, 1221, 1235, 1355, 1453, 1509, and Adonis), the distances of which vary between a = 1.85 and a = 1.98. This means that in relation to Mars their q-values are comprised between the limits

the mean of all being

$$q = 1.26$$
.

This is only a little more than was anticipated $(q \approx 1.2)$.

After identifying the group mentioned with Ultramars, there is only one asteroid left inside the *normal asteroids*. This is the very faint object Hermes. Its distance is a=1.29, which gives (in relation to the Earth) q=1.29. It thus lies not far from where Ultratellus would be predicted. Its inclination is small (i=4.7), but its eccentricity is very large $(\varphi=28^{\circ})$. It may be possible to identify Hermes with Ultratellus.

According to § 5 Infrajupiter might be predicted at a distance of about 4.3. In this region there is a single asteroid, 79 Thule, with a = 4.25. This gives in relation to Jupiter

$$q = 1.22$$

which is in good agreement with the value expected in § 5. Its inclination, as well as its eccentricity, is low $(i > 2.3^{\circ}, \varphi = 3.5^{\circ})$.

On the other hand there is a group containing 19 asteroids (153, 190, 334, 361, 499, 748, 958, 1038, 1162, 1180, 1202, 1212, 1256, 1268, 1269, 1345, 1439, 1512, 1529) situated at 3.90 < a < 4.00, which gives

$$1.30 < q < 1.33$$
.

This is somewhat higher than expected, but the asteroids identified as Ultramars and Ultratellus (?) also gave q-values a little higher than expected.

The period of this group is commensurable with that of Jupiter. The same holds for Thule. This shows the strong influence of the resonance with Jupiter in this region. As is known from the Saturnian inner satellites, such a resonance can displace the situation of a body considerably. It is possible that the Infrajupiter group has been split up because of this, so that Thule, together with the 3.95 group, has to be counted as Infrajupiter.

The Infrajupiter group is characterized by small inclination ($i < 13^{\circ}$) and rather low eccentricity ($\varphi < 14^{\circ}$).

Accepting the above identifications, we obtain three new dots in the q—Q-diagram. As the Q-values of these are close to o or ∞ , the logarithmic plot is not appropriate. If we use instead a scale proportional to $\frac{Q}{1+Q}$, we obtain the diagram shown in Fig. 6. The three dots containing Ultratellus, Ultramars, and Infrajupiter lie satisfactorily in relation to the curves.

Of the asteroids there are only three bodies still unclassified: 522 Helga, 1144 Oda, and 944 Hidalgo. Of these, Helga and Oda are situated between the normal asteroids

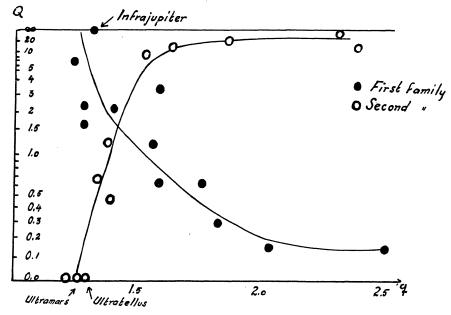


Fig. 6. q-Q diagram including Infrajupiter, Ultramars, and Ultratellus.

and the Infrajupiter group. As most asteroids have rather eccentric orbits, the normal asteroids and the Infrajupiter group move in part in the same region. Consequently there is a certain probability that a member of the normal asteroids will collide with a member of the Infrajupiter group, and a rough calculation shows that during the life of the planetary system this probability has been considerable. As the collision must be inelastic, it will result in intermediate values of the mean solar distance a of both bodies. In this way Helga and Oda, perhaps once members of the normal and of the Infrajupiter group, may have attained their present positions. As the result of such a collision, the eccentricity as well as the inclination of the orbits usually decrease (compare Lindblad's theorem). Thus, we should expect Oda and Helga to have rather small values of i as well as of φ . Helga has $i = 4.4^{\circ}$ and $\varphi = 4.9^{\circ}$, whereas the values for Oda are $i = 9.7^{\circ}$ and $\varphi = 4.8^{\circ}$. Compared with the usual values for asteroids, these inclinations are about normal, but the eccentricities must be considered as rather small.

If the positions of 522 Helga and 1144 Oda are explained in this way, there remains only one asteroid of the known 1542 to be accounted for. This asteroid is 944 Hidalgo. Its mean distance is a = 5.80, inclination $i = 42^{\circ}$, and angle of eccentricity $\varphi = 41^{\circ}$. This small planet (diameter = 50 km.) is the only body in the solar system which cannot be brought into our general scheme of classification.

It is of some interest to discuss the relative ages of the different asteroid groups.

As the Ultramars group is situated in the »shadow» of the normal asteroids, it cannot Stockholms Observatoriums Annaler. Band 14. N. o 9.

have been formed after them. This is in agreement with the general scheme of the formation, according to which the Moon—Mars group, to which Ultramars belongs, is the oldest part of the whole system.

The Infrajupiter group must, of course, have been formed immediately after Jupiter. As the outer limit of the normal asteroids is given by Jupiter's shadow (compare I § 16), these must have originated after Jupiter.

It is not possible to decide whether the Ultramars or the Ultrajupiter group is the elder, so the analysis of the asteroids does not help us to settle the question whether the Moon—Mars group is older than the giant planets (as is probable for other reasons).

The discussion of the asteroids has thus given the following result. Most of the asteroids constitute the "asteroid ring" (normal asteroids) between 2.2 and 3.4, which can be considered as an analogy to the Saturnian ring. Outside the normal asteroids are found the Trojans and two other large groups, which are situated just where Infrajupiter and Ultramars are expected. Two asteroids have about the same distance as Mars and are classified as "Martian Trojans". Then only five asteroids remain. One of these may be Ultratellus, one may belong to the Ultrajupiter group and two are tentatively thought to be due to a collision between the normal and the Infrajupiter groups. One is left, impossible to account for.

D. The satellite systems.

With the help of the classification made in this paper and the theory proposed in I and II, we shall here discuss the satellite systems.

§ 7. First-family satellites.

Only Jupiter and Saturn are large enough to have first-family satellites. The masses of all other planets are so small that the gravitational potentials at their surfaces are smaller than the limiting value at which the gas becomes ionized and stopped according to I. Uranus and Neptune are very close to this limit — on which side, it is difficult to say — so they may have rudiments of a first-family satellite system.

The Martian satellite system is remarkable. Its q-Q-dot falls on the first family curve, but the gravitational potential of the satellites is very much smaller than that characteristic of the first family. Only the abnormal satellites Jupiter VIII, IX, and XI, and Saturn IX (Phoebe) have such small values. There is no reason, however, to believe that the Martian satellites are abnormal, because their orbits have small eccentricities (e = 0.017 and 0.003) and small inclinations. Consequently, as far as is known, there is no real

analogy to the Martian system elsewhere, but as Mars is supposed to be the oldest member (except the Moon) of the whole system, its satellites may be a reminiscence of an event that happened before the formation of the other planets.

It is naturally possible to extrapolate the satellite groups in the same way as the planetary groups.

§ 8. Second-family satellites.

Second-family satellite systems are found around Saturn and Uranus. The gravitational potential of their members varies between 1.63 and 4.77 · 10¹⁸ for the Saturnian and between 1.62 and 4.95 · 10¹⁸ for the Uranian system.

The masses of Jupiter, Neptune, and of the Earth are large enough to permit the existence of systems with this energy.

For the Earth, a gravitational potential of $4.9 \cdot 10^{18}$, which is the upper limit for the energy in the Saturnian and Uranian systems, corresponds to a distance of $1.22 \cdot 10^9$ cm, which is almost twice the Earth's radius. Thus, the lower limit of a second-family satellite system around the Earth would be situated well above the Earth's surface but below the Roche limit (= 2.44 times the radius). Consequently, we should expect a system similar to the Saturnian system consisting of rings and a few small satellites.

The formation of such a system, however, would be very much disturbed by the Moon. The mass of the latter is as much as 0.012 of that of the Earth, which means that the centre of gravity of the Earth—Moon system is situated 0.47·109 cm from the centre of the Earth. This is almost 40% of the lower limit of the supposed satellite system. It is obvious that the motion of the particles which were about to form the system must be greatly changed by the presence of the Moon. This may be the explanation for the non-existence of these satellites.

Neptune also has a large abnormal satellite which may have prohibited the formation of a normal satellite system.

The same argument cannot be applied to the Saturnian system, because the giant satellite Titan belongs itself to the second family. Further, counted in relation to its primary, its mass is only $\frac{1}{50}$ of the Moon's. Besides, the rest of the second-family satellite system of Saturn has really been disturbed, as indicated by the fact that there is a very small satellite, Themis, which is irregular and forms a sort of doublet to Hyperion, which may be considered as normal.

The reason why Jupiter has no second-family satellites is not quite so clear. It certainly has three abnormal satellites (VI, VII, and X) in just that region where such a

system is to be expected. (The large eccentricities and inclinations of these satellites and the irregular sequence of their distances makes it impossible to count them as normal satellites.) These, however, are so small that it is unlikely that they could have prohibited or disturbed the formation of a normal satellite system.

It is possible that the magnetic field of Jupiter at the time of formation was too weak, so that at this distance the solar magnetic field disturbed the motion of the ionized matter excessively, but this cannot be more than a guess, since we do not know the actual strength of the magnetic field, and far less the strength at the time of formation.

In support of this guess the following remarks may be made. It is resonable to assume that the magnetic dipole moments of the giant planets were approximately proportional to their masses. Let us put them equal to

$$a_n = \alpha \cdot M_n$$

where α is a constant and M_n the mass of a planet at the solar distance $= r_n$. Suppose further that the Sun's magnetic moment was α_0 . The ratio between the magnetic fields of the Sun and that of the planet at the distance ϱ_n from the planet is

$$\eta = rac{a_0}{lpha} \quad rac{{arrho_n}^3}{M_n \, {r_n}^3}$$

If we put the gravitational potential of the second family bodies equal to

$$G = \frac{M_n}{\varrho_n}$$

we find

$$\eta = \frac{a_0}{\alpha G^3} \quad \frac{M_n^2}{r_n^3}$$

The values of $M_n^2 r_n^{-3}$ for the planets considered are:

	$m{M}$	r	$M^2 r^{-3}$
Jupiter	317	5.20	710
Saturn	95	9.54	. 10
$\mathbf{U}\mathbf{ranus}$	14.5	19.19	0.03

Thus, if the Sun's magnetic field equalled Jupiter's in the region of the second family it was only $\frac{10}{710}$ of the Saturnian field in the same region, and still less in the Uranian system. Consequently, the Jovian system is most sensitive to such a disturbance, and if the Sun's magnetic field has prevented the formation of a second-family system around Jupiter, it need not have affected the Saturnian and Uranian systems appreciably.

Of the normal satellites in the solar system there is only one tiny member left unclassified, the fifth satellite of Jupiter. To judge from its gravitational potential $\left(\frac{M}{r}=1.05\cdot 10^{18}\right)$ it could be a counterpart to the inner planets. The meteorites responsible for the second family may very well have produced planets in the neighbourhood of Jupiter as well as in the environment of the Sun.

§ 9. Abnormal satellites.

The characteristic features of abnormal satellites are that their orbits have relatively large eccentricities and inclinations (towards the equatorial planes of the planets). In some cases they move in the retrograde direction.

We know the following abnormal satellites: The Moon, six Jovian satellites VI—XI, the Saturnian satellite IX, and the Neptunian satellite Triton.

As we have seen, the Moon, and perhaps also the Neptune satellite Triton, may be counted as members of an adjacent planetary group, which have been captured. Probably the Moon, which forms a group together with Mars, existed before the Earth, and when the latter was formed, it captured the Moon. A similar process may have been occurring between Neptune and Triton.

The Jovian satellites VIII, IX, and XI have a retrograde rotation and are very loosely bound. It is generally supposed that they are captured asteroids.

The satellites VI, VII, and X may also have been captured although they are much more closely bound. Their rotations are direct.

The Saturnian satellite Phoebe has a retrograde rotation. Even this may be a captured asteroid, although only a few asteroids are known which may approach the Saturnian orbit. As we have seen there are asteroids with distances almost equal to those of Jupiter and Mars and there seems to be no reason why Saturn should not have — or have had — one with the same property.

If we wish to systematize the abnormal satellites, we are not interested in their gravitational potentials, because this parameter has no significance in the capture process through which they probably attained their present positions. Instead, the correct procedure must be to relate their distances from the primaries to the unit of length defined in II $\S 9$, since this gives a measure of how strongly they are bound. For example, at a distance of $\sqrt[3]{\frac{1}{3}}$ times this unit in the direction towards the Sun, there is a libration point where a particle can move at a fixed position in relation to the planet as well as to the Sun.

The unit of length l was defined by

$$l = r_1 \sqrt[3]{\frac{\overline{M_1}}{\overline{M_0}}}$$

where r_1 is the solar distance and M_1 the mass of the planet, and M_0 the mass of the Sun. In relation to l, all phenomena regarding the interference between the gravitational fields of the sun and a planet must be the same for all planets.

The values of l for different planets are given in Table 2.

Table 2.

Planet:	,Earth	Mars	Jupiter	Saturn	Uranus	Neptune
<i>l</i> ⋅ 10 ⁻¹¹ cm	2.16	1.56	77	94	101	167

If l is used as unit of length, the distances of the abnormal satellites are:

Table 3.

Planet	Satellite	Distance
Earth	Moon VI VII	0.178 0.149 0.152
	X	0.154
	VIII	0.306
	IX	0.313
	XI	0.292
Saturn	Phoebe (IX)	0.137
Neptune	Triton	0.0021

The reduced distance is about the same for the Moon, Jupiter VI, VII, X, and Phoebe. Whether this has a significance or not can only be ascertained when a theory for the supposed capture is developed. It may be possible that, in the connection with the formation of the planets, a certain type of capture has proceeded, leading to abnormal satellites at a distance of about 0.15. In the case of Jupiter VI, VII, X, and Phoebe there has been no tidal action to change the distance. The Moon's distance is increased a little by the tides and the retrograde Triton has been braked so much in the same way that its distance is now very much smaller. If Jupiter VIII, IX, and XI are considered as asteroids more recently captured, we might account for all the abnormal satellites in this way.

Non-solar planets.

§ 10. The theory presented in I starts from the assumption that the Sun once passed a nebula (containing gas and dust particles). When the matter of the nebula fell in towards the Sun, those processes which resulted in the formation of the solar system occurred. It would also be possible to imagine that no actual passage of a nebula took place, but that the formation of the planets was intimately connected with the formation of the Sun itself. According to current views the Sun was formed from a primordial nebula. When the Sun had just been formed, some remnants of this nebula may have fallen in, giving rise to the formation of the planets. It is difficult to judge which of these hypotheses is to be preferred. The latter has the advantage that a far-reaching analogy between the formation of the Sun and planets on the one hand and the planets and their satellites on the other is obtained.

According to the latter hypothesis we should expect all stars to be surrounded by planets. If the former hypothesis is preferred, at least a considerable number of them are likely to have planets, because the probability of the passage of a star through a nebula is rather high.

It is reasonable to assume that the chemical constitution of the nebulae, which have caused formation of planets (according to the former or the latter hypothesis), is essentially the same everywhere. This implies, according to our theory, that the gravitational potential of the planets is the same, irrespective of the star around which they are formed. Consequently, all planetary systems are likely to have the same constitution as ours, so that the relative distances and the relative masses are about the same. The absolute values of the distances are uniformly enlarged or diminished in proportion to the mass of the central star, and the absolute values of the planetary masses depend on the total mass captured from the planetforming nebula. For example, around a star with the mass M times the solar mass there would be a planet analogous to Jupiter, having the same gravitational potential as Jupiter and consequently moving in an orbit M times that of Jupiter, i. e. at the distance

 $a = 5.2 \cdot M$ astr. units.

Its mass might be smaller or bigger than Jupiter's, depending upon the total mass of gas captured by the star.

The discovery of a couple of non-solar planets makes is possible to check this consequence. Disturbances of the orbit of the double star 61 Cygni indicate that at least

¹ A. Hunter, Nature 152, p. 66, 1943. — K. A. Strand, Astr. Soc. Pac. 55, p. 29, 1943. — D. Reuyl and E. Holmberg, Astrophys. J. 97, p. 41, 1943. — H. Alfvén, Nature 152, 1943.

one of its components has at least one planet. As the masses of the stars are about equal (0.55 and 0.58 solar masses) it does not matter to which of them the planet belongs. According to our theory the planet systems of the stars ought to have the same construction as the solar system. If this is so, the disturbance must obviously be due to that planet which is analogous to Jupiter, the largest planet. Its theoretical orbital radius is

$$a = 5.2 \cdot 0.565 = 2.9$$

if the average 0.565 of the masses is used. The observational value is

$$a = 2.4$$

which must be considered as being in good agreement.

There are also good reasons to believe that at least one of the components of the double star 70 Ophiuchi has a planet. In this case the masses of the components are different (1.1 and 0.7). From observations the distance to the planet is

$$a = 6.8$$
 or $a = 5.9$

depending upon whether it belongs to the heavy or the light component. The theoretical distances to the biggest planet in the system are for the two cases

$$a = 5.2 \cdot 1.1 = 5.7$$
 or $a = 5.2 \cdot 0.7 = 3.6$.

From observations it has not been possible to decide to which component the planet belongs. From the theoretical point of view it is likely that the heavier component has captured more mass from the supposed nebula than the lighter component. In fact we should expect the mass captured by a star from a nebula to be proportional to the square or to the cube of the mass of the star, ceteris paribus. This means that the disturbance of the position of the star due to a planet analogous to Jupiter ought to be the larger the larger the star. Thus, it is theoretically most likely that the detected planet belongs to the heavier component. For this case the observational distance is a = 6.8 and the theoretical distance a = 5.7. The agreement is as good as can be expected.

Consequently, we are justified in saying that all the five first-family systems we know (the planetary system, the Jovian and Saturnian satellite systems, and the two non-solar systems) are built according to the same laws. For all of them the gravitational potential has about the same value, as is seen from the following table.

¹ This could be compared whith the conditions in the satellite systems: The ratio between the total masses of the first-family satellites of Jupiter and of Saturn is about 75. The ratio between the masses of the two planets is 3.3. The ratio between the total masses of the second-family satellites of Saturn and Uranus is about 9, whereas the ratio between the planetary masses is 6.5. In both cases the bigger planet has the larger total satellite mass.

Table 4.

Group	Gravitational potential · 10 ⁻¹⁸
First-family planets	4.4—25
» » Jovian sat	10-45
» » Saturnian sat	11-31
61 Cygni planet	31
70 Ophiuchi »	2 I

The value for Jupiter, which is to be compared with the non-solar planets, is 25. If the planet of 70 Ophiuchi belong to the lighter component, as is improbable but not impossible, its value would be 15.

§ 11. Remarks on the transfer of angular momentum.

One of the most important questions in connection with the origin of the solar system is how the planets have acquired their angular momenta. According to the theory developed in I and II, a transfer of momentum from the sun to the planets has been effected by electromagnetic forces. In an earlier paper¹, it was shown that if the Sun were surrounded by a conducting medium (e. g. an ion cloud), its rotation in combination with its general magnetic field would produce currents strong enough to change its angular momentum considerably during even so short a time as 10⁵ years. This result was obtained under the assumption that the transfer of momentum was effected by electric currents produced through unipolar induction and limited by the condition that the magnetic field produced by them must not be stronger than the general solar field causing them.

There are two objections against the application of this result to the acceleration of an ion cloud at the distance of Jupiter from the Sun. The first is that the solar magnetic field near Jupiter's orbit is very much weaker than close to the solar surface, which sets a much lower limit to the transfer of momentum.

The other objection — which works in the other direction — is that the acceleration ought to be effected by means of magneto-hydrodynamic waves², and if so there is no reason why the limit to the accelerating force should be set so low as by the above condition. According to a recent theory, the sunspots are due to a new type of wave, called magneto-hydrodynamic waves, which are produced through convection near the Sun's centre and are transmitted along the magnetic lines of force of the general

¹ H. ALFVÉN, Remarks on the rotation of a magnetized sphere with application to solar rotation, Ark. f. mat. astr. o. fysik Bd 28 A, No 6, 1942.

H. Alfvén, Ark. f. mat. astr. och fysik Bd 29 B, No 2, 1942; Nature 150, p. 405, 1942; M. N. 105, p. 3, 1945.
 Stockholms Observatoriums Annaler. Band 14. N:0 9.

magnetic field out to the surface, where they cause sunspots. The waves transfer a state of rotation from the solar core to the surface. From observational evidence, it is obvious that the maximum angular momentum transferred is not limited by the condition that the magnetic field from the induced currents must not surpass the initial field. In fact, the induced field, which is the field of the sunspots, is several thousand gauss, whereas the general magnetic field is only about 10 gauss. According to Walén the field of the magneto-hydrodynamic waves amounts to 10,000 gauss at a depth of 2.5·10⁹ cm below the solar surface¹, so that the induced field is 1,000 times stronger than the primary field. This implies that the angular momentum transmitted is 1,000 times as large as it would be if limited by the above condition. There is no reason to believe that this represents an upper limit.

Magneto-hydrodynamic waves can be produced in any conducting medium if a magnetic field is present. Thus, in a medium consisting of the Sun and an ion cloud surrounding it and affected by the solar magnetic field, such waves could be generated. In fact, any unequality in the state of rotation of different parts of the medium (e. g. between the Sun and the ion cloud) will produce waves which tend to equalize the angular velocities of different parts of the medium. A transfer of momentum from the Sun to the cloud (which later forms the planets) can be produced in this way. The theory of the magneto-hydrodynamic waves is not so developed that it is possible to say what provides the upper limit to the transfer of momentum, but it is quite possible that this limit is several powers of ten above that given in the cited paper.

Furthermore, it must be observed that the magnetic dipole moment of the Sun may have been much greater at the time of the formation of the planets than it is now. There is no general agreement as to the cause of the solar magnetic field but there are some good arguments in favour of Elsasser's theory. According to this, currents in the Sun's interior are produced by thermoelectric forces. If this is correct, the dipole moment should be proportional to the thermoelectric current and to the surface of the circuit. The current is proportional to the electromotive force, to the conductivity, and to the solar radius, and the surface of the circuit is proportional to the square of the solar radius (R). Consequently, if the electromotive force and the conductivity are constant, the dipole moment is proportional to R^3 . If the Sun were 100 times larger at the time of the formation of the giant planets, as might be possible, its dipole moment may have been 10^6 times greater than now.²

¹ C. Walén, Ark. f. mat. astr. och fysik Bd 30 A, No 15, 1944, p. 52.

² COWLING (M. N. 105, p. 166, 1945) has recently developed ELSASSER's theory and shown that in his modification it can explain only 10⁻⁷ of the Sun's magnetic field. It may be possible, however, that some

The result of our brief discussion in this paragraph is that the mechanism of acceleration as outlined in the cited paper does not give enough momentum to the giant planets with the present value of the solar magnetic field. An acceleration through magneto-hydrodynamic waves, however, is likely to give an acceleration several powers of ten higher. Moreover, we cannot exclude the possibility that the solar dipole moment at the formation of the planets was several powers of ten times the present value. Neither the theory of the magneto-hydrodynamic waves nor the theory of the general solar magnetic field is at present in such a state as to allow us to decide whether the process of acceleration was quantitatively possible or not within a reasonable length of time.

§ 12. Conclusions.

- It is obvious that this theory, as well as all other theories of the origin of the solar system includes many speculative factors. It is therefore important to find what may be considered as rather well established facts. In my opinion the most certain results of this paper and I and II are the following:
- I. The planetary system and the satellite systems have been formed in the same way. From the classification in §§ 1—4, it is evident that the q—Q diagrams and the rules of the gravitational potential are applicable in the same way to the planetary system, the Jovian, Saturnian, and Uranian, (and perhaps in part also to the Martian) satellite systems. This can be understood only if these systems have been formed in the same way. It is also probable that the two non-solar planets discovered obey the same laws.
- 2. The planets have been formed of matter falling in from interstellar space towards the Sun. The satellites have been formed of matter falling in from interplanetary space towards the planets.

The matter forming the planets (satellites) must have originated either from the Sun (mother planet) or from interstellar (interplanetary) space. As is shown in §§ 1—4, the gravitational potential is of fundamental importance. This quantity means the difference in energy between the present position of a body and the same body placed at infinite distance. If the matter has fallen in from space, it is quite natural that this quantity is of importance. On the other hand, if the planets were expelled from the Sun (and the satellites from their planets) we should instead expect the difference in potential

other modification of the theory is more tenable. Cowling discusses the possibility that the solar magnetic field is a relict of a field generated in connection with the birth of the Sun. If this is correct, and if we accept the hypothesis tentatively suggested at the beginning of § 10 that planets were formed immediatly after the Sun, it would also be reasonable to assume that at the formation of the planets the field was much stronger than now.

between the Sun's (planet's) surface and the present position of the planet (satellite) to be of importance, but it would be very difficult to understand how the absolute value of the gravitational potential could enter.

- 3. At least two processes of different kinds have occurred, leading to the formation of the first and the second family.
- 4. With regard to masses and orbital radii, the solar system is essentially in the same condition now as immediately after its formation.

The relations between masses and orbital radii, expressed in the q-Q diagrams, must be due to laws governing the formation of the planets and satellites. The regularity of the diagrams indicates that no major change has occurred.

There are also good arguments for the following conclusion, although it must be considered as less certain than those above:

5. Before its condensation the matter forming the planets (satellites) has been ionized and supported, at least in part, by the Sun's (mother planet's) magnetic field.

The »shadows» occurring in two places in the Saturnian ring system and in the corresponding two places in the asteroid ring confirm the »two-thirds law» which is derived under the assumption that ions supported by the solar magnetic field recombine as the first step of the condensation process. The q-Q diagrams can be understood, at least qualitatively, on the same assumption (see II §§ 1—8).

Summary.

In §§ 1—4 a classification of the planets and satellites is made without reference to any theory. It proves that a division into two distinct »families» can be made on three different grounds: the density, the gravitational potential, and the position in the q-Q diagram. The bodies of the first family have densities below 3.0, a gravitational potential within the limits $4.5 - 50 \cdot 10^{18}$ and q-Q dots situated on the falling curve. The second family is characterized by a high density (> 3.0), a gravitational potential below $4.5 \cdot 10^{18}$ or above $50 \cdot 10^{18}$, and by a rising q-Q curve.

The regularities found (especially in the q-Q diagram) make possible an extrapolation to unknown planets (§ 5). Two or three of these are identified with groups of asteroids.

For the classification of the asteroids, it is advantageous to derive some help from the theory developed in two preceding papers. It is shown that the asteroids belong to the »asteroid ring», which is analogous to the Saturnian ring, or to some of the adjacent planetary groups (§ 6).

The laws found can be applied to the planetary as well as to the satellite systems (§§ 7-9) and also — as is pointed out in § 10 — to the non-solar planets newly discovered.

At last some remarks are made about the transfer of momentum from the Sun to the planets, which is fundamental to the theory (§ 11). The importance of the magneto-hydrodynamic waves in this respect is pointed out. It is possible that the Sun's general magnetic field was much stronger at the time of formation of the solar system than it is now. This must be assumed in order to explain the momentum transfer quantitatively.

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