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Abstract. If viscosity is taken into account, Keplerian motion of a large number of grains in a gravitational field has a tendency to lead to the formation of jet streams.

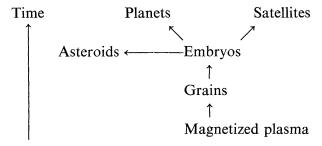
In order to treat problems of this kind it is advantageous to present celestial mechanics by a simple perturbation approach which is developed in Sections 2–7.

Inelastic collisions between a number of grains will tend to make their orbits similar. This leads to the formation of jet streams. Their properties are treated in Sections 8–10.

Finally, in Section 11, we discuss the possible application of the jet stream theory to meteor streams, to asteroidal jet streams, and to the cosmogonic accretion process.

1. Cosmogonic Importance of Jet Streams

The study of the origin and evolution of the solar system involves the clarification of a number of processes which are connected in the following way (Alfvén, 1967a).



The process by which the magnetized plasma is injected in the neighbourhood of a central body depends on the 'critical velocity', a phenomenon which has been studied extensively (cf. Alfvén, 1960; Fahleson, 1961; Eninger, 1966).

The transfer of angular momentum from the central body has been treated in a recent paper (Alfvén, 1967b) and seems to be associated with a special type of corotation ('partial corotation'). A condensation from a partially corotating plasma produces 'grains' moving in orbits with eccentricity $e=\frac{1}{3}$. They accrete to larger bodies called 'embryos', which by further accretion grow to satellites, if the process takes place around a planet, or to planets around the sun.

The accretion is associated with a serious difficulty. The state which is an immediate result of the condensation is similar to the present state in the asteroid belt in the sense that in general the grains collide with relative velocities of the order of $10^5 - 10^6$ cm sec⁻¹. It is well known that such collisions ('hypervelocity' collisions) usually lead to fragmentation and not to accretion. Only if the relative velocities are brought down to well below 10^5 cm sec⁻¹ will an accretion take place.

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This difficulty is intrinsic to all 'planetesimal' theories. As there are convincing arguments for the planetesimal approach, the only way out seems to be that the accretion occurs in two steps: The first step is that the grains form a number of *jet streams* in interplanetary space. (By jet streams we mean that a large number of bodies move in almost exactly the same orbit.) Inside such a stream the relative velocities decrease due to inelastic collisions until they have reached such a low value that satellite or planetary accretion begins.

The purpose of this paper is to give a preliminary discussion of the possible formation of jet streams and of their properties. Although in principle a jet stream consisting of electrically charged grains may be kept together by electromagnetic effects, it seems more likely that the formation of jet streams can be understood as a simple result of viscous effects acting on grains moving in Kepler orbits.

It seems possible that jet-stream producing processes are active even today. Some meteor streams and the jet streams recently discovered in the Hirayama families (Alfvén, 1968; Arnold, 1969; Danielsson, 1969) may be examples of this.

There is also a possibility that jet-stream formation may be an intermediate stage in star formation.

2. Different Presentations of Celestial Mechanics

In order to study jet streams we must use celestial mechanics. The traditional way of presenting this field of science has the aim of making it useful for the preparation of the Nautical Almanac, and nowadays also for the calculation of spacecraft trajectories. This way is not very suitable if we want to study the interaction of grains with a plasma or with any viscous medium, or the mutual interaction between grains. It is more convenient to use an approximate method, the essence of which is that an elliptical orbit is treated as a perturbation of a circular orbit. This method is useful for orbits with small eccentricities, but breaks down when the eccentricity is large. From a formal point of view the method has some similarity to the guiding-centre method of treating the motion of charged particles in a magnetic field (Alfvén and Fälthammar, 1963). This means that ordinary celestial mechanics corresponds to the Störmer method. For high-energy particles only the Störmer method is applicable but for small energies the guiding-centre method, although approximate, is much more convenient. Similarly, for orbits with eccentricities approaching unity the approximate method is not applicable, but when the eccentricity is small it is often much simpler.

3. Circular Orbits

If a body with negligible mass is moving around a central body, an important quantity is the angular momentum C (per mass unit) of the small body with reference to the central body (or, strictly speaking, to the centre of gravity). This is defined as

$$\mathbf{C} = \mathbf{r} \times \mathbf{v},\tag{1}$$

where \mathbf{r} is its distance to the centre, and \mathbf{v} is its orbital velocity. \mathbf{C} is an invariant vector during the motion.

The body is acted upon by the gravitational attraction f_g of the central body and by the centrifugal force

$$f_c = \frac{v_\psi^2}{r} = \frac{C^2}{r^3} \,, \tag{2}$$

where v_{ψ} is the tangential velocity component.

The simplest type of motion is the motion with constant velocity v_0 in a circle with radius r_0 . The gravitational force f_g is exactly compensated by the centrifugal force, $f_c = f_g$.

We have

$$v_0 = \frac{C}{r_0} = (r_0 f_g)^{1/2} = \frac{r_0^2 f_g}{C}.$$
 (3)

The angular velocity of the motion is

$$\omega_K = \frac{v_0}{r_0} = \left(\frac{f_g}{r_0}\right)^{1/2} = \frac{r_0 f_g}{C} = \frac{C}{r_0^2} \tag{4}$$

with the period $T_K = 2\pi/\omega_K$.

4. Oscillation

The body can perform oscillations around this orbit in both the radial and axial directions.

If the body is displaced radially from r_0 to $r = r_0 + \Delta r$, it is acted upon by the force

$$\Delta f_r = f_c - f_g = \frac{C^2}{r^3} - f_g(r).$$
 (5)

Because the force is zero for $r = r_0$ we obtain

$$\mathrm{d}f_r = -\left(\frac{3C^2}{r^4} + \frac{\partial f_g}{\partial r}\right)\mathrm{d}r\,. \tag{6}$$

This means that the body oscillates around the circle with the angular frequency

$$\omega_{r} = \left(-\frac{\mathrm{d}f_{r}}{\mathrm{d}r}\right)^{1/2} = \left(\frac{3C^{2}}{r^{4}} + \frac{\partial f_{g}}{\partial r}\right)^{1/2} = \left(\frac{3f_{g}}{r_{0}} + \frac{\partial f_{g}}{\partial r}\right)^{1/2}.$$
 (7)

If it is displaced in the z-direction (axial direction), it is acted upon by the force f_z , which because of div f = 0 is given by

$$\frac{\partial f_z}{\partial z} = -\frac{1}{r} \frac{\partial}{\partial r} (r f_g) = -\frac{1}{r} \left(f_g + r \frac{\partial f_g}{\partial r} \right). \tag{8}$$

The angular velocity of the oscillation is

$$\omega_z = \left(-\frac{\partial f_z}{\partial z}\right)^{1/2} = \left(-\frac{f_g}{r_0} - \frac{\partial f_g}{\partial r}\right)^{1/2}.$$
 (9)

From (7), (9), and (4) we find

$$\omega_r^2 + \omega_z^2 = 2\omega_K^2. \tag{10}$$

We place a moving coordinate system with the origin in a point moving along the circle r_0 with the angular velocity ω_K (Figure 1). The x-axis points in the radial direction and the y-axis in the forward tangential direction. We have

$$x = r\cos(\psi - \omega_K t) - r_0, \tag{11}$$

$$y = r\sin(\psi - \omega_K t), \tag{12}$$

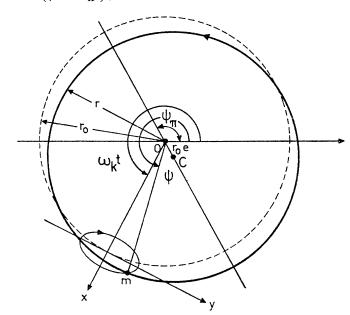


Fig. 1. The central body is at the origin O, which is the centre of the broken (----) circle with radius r_0 . Radial oscillations change the orbit of a point mass m into an ellipse, which almost coincides with the circle (——) which has its centre at C. The distance OC is r_0e . The position of the pericentre is given by ψ_{π} . The difference between the full circle and the exact Kepler ellipse is really less than the thickness of the line. Let the origin of a coordinate system (x, y) move with constant velocity along the broken circle. In this coordinate system the point mass m moves in an 'epicycle' which is an ellipse with the axis ratio 2:1. The epicycle motion is retrograde.

where ψ is the angle measured from a fixed direction. The radial oscillation can be written as

$$r = r_0 \left[1 - e \cos \left(\omega_r t - \psi_r \right) \right], \tag{13}$$

where er_0 is the amplitude $(e \le 1)$ and $\psi_r - a$ constant. Because C is constant, we have

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = \frac{C}{r^2} \approx \frac{C}{r_0^2} \left[1 + 2e\cos(\omega_r t - \psi_r) \right]. \tag{14}$$

As $x \ll r_0$ and $y \ll r_0$ we find from (11), (12), (13), (14), and (4):

$$x \approx r - r_0 = -r_0 e \cos(\omega_r t - \psi_r)$$

= $-r_0 e \cos(\omega_K t - \omega_\pi t - \psi_r)$, (15)

$$\frac{\mathrm{d}y}{\mathrm{d}t} = r_0 \left(\frac{\mathrm{d}\psi}{\mathrm{d}t} - \omega_K \right) = \frac{2eC}{r_0} \cos(\omega_r t - \psi_r), \qquad (16)$$

or after integration

$$y = 2r_0 e \left(1 + \frac{\omega_{\pi}}{\omega_r}\right) \sin(\omega_K t - \omega_{\pi} t - \psi_r), \tag{17}$$

where we have put

$$\omega_{\pi} = \omega_{K} - \omega_{r}. \tag{18}$$

The pericentre (point closest to the centre) is reached when x is a minimum, that is when

$$\omega_K t - \omega_\pi t - \psi_r = 2\pi n \quad (n = 0, 1, 2...). \tag{19}$$

If we put

$$\psi_{\pi} = \omega_{\pi} t + \psi_{r}, \tag{20}$$

the pericentre is reached when $t = (\psi_{\pi} + 2\pi n)/\omega_{K}$. This shows that the pericentre moves (has a 'precession') with the velocity ω_{π} , given by (18).

In a similar way we find the axial oscillations

$$z = r_0 i \sin(\omega_z t - \psi_z) = r_0 i \sin(\omega_K t - \omega_\theta t - \psi_z), \tag{21}$$

where $i (\leq 1)$ is the inclination and

$$\omega_{\theta} = \omega_{K} - \omega_{z}. \tag{22}$$

The angle ψ_{θ} of the 'ascending node' (point where z becomes positive) is given by

$$\psi_{\theta} = \omega_{\theta} t + \psi_{z}. \tag{23}$$

5. Coulomb Force

If the mass of the orbiting body is taken as unity, then

$$f_g = \frac{\mu}{r^2} \tag{24}$$

with $\mu = \kappa M_c$, where M_c is the mass of the central body, and κ the gravititional constant. As $f_c = f_g$ for the undisturbed motion we have from (2) and (24)

$$C = (\mu r_0)^{1/2} \,. \tag{25}$$

From (24) we get

$$\frac{\partial f_g}{\partial r} = -\frac{2f_g}{r}. (26)$$

Substitution of (26) into (7) and (9) shows that

$$\omega_r = \omega_z = \omega_K \tag{27}$$

with

$$\omega_K = \frac{\mu}{Cr_0} = \left(\frac{\mu}{r_0^3}\right)^{1/2}.$$
 (28)

The significance of (27) is that the frequencies of radial and axial oscillation coincide with the fundamental angular velocity of circular motion. Consequently we have $\omega_{\pi} = \omega_{\theta} = 0$ and there is no precession of the pericentre or of the nodes. According to (15) and (17) the body moves in the 'epicycle'

$$x = -r_0 e \cos(\omega_K t - \psi_r),$$

$$y = 2r_0 e \sin(\omega_K t - \psi_r).$$
(29)

The centre of the epicycle moves with constant velocity along the circle r_0 . The motion in the epicycle takes place in the retrograde direction. See Figure 1.

The axial oscillation can be written

$$z = r_0 i \sin(\omega_K t - \psi_z). \tag{30}$$

We still have an ellipse but its plane has the *inclination* i with the plane of the undisturbed circular motion. The axial oscillation simply means that the plane of the orbit is changed from the initial plane, which was arbitrarily chosen because in a Coulomb field there is no preferred plane.

6. Motion in the Field of a Rotating Central Body

According to (27) the motion in a Coulomb field is degenerate in the sense that $\omega_r = \omega_z = \omega_K$. This is due to the fact that there does not exist any preferred direction.

In the planetary system and in the satellite systems the motions are *perturbed* because the gravitational fields deviate from Coulomb fields. This is essentially due to the effects discussed in this section and in Section 7.

The axial rotations (spins) of the planets change the shape of these from spherical to ellipsoidal. We can consider their gravitation to consist of a Coulomb field from a sphere, on which is superimposed the field from the 'equatorial bulge'. The latter contains higher order terms, but has the equatorial plane as the plane of symmetry. We can write the gravitational force in the equatorial plane

$$f_g = \frac{\mu}{r^2} \left(1 + \frac{\alpha}{r^2} \right) \tag{31}$$

taking account only of the first term from the equatorial bulge. The constant α is

always positive. As in this case

$$\left|\frac{\partial f_g}{\partial r}\right| > \frac{2f_g}{r} \tag{32}$$

we have from (7), (9), and (4)

$$\omega_z > \omega_K > \omega_r$$
. (33)

According to (18) and (22) this means that the pericentre move with the velocity

$$\omega_{\pi} = \omega_{K} - \omega_{r} > 0 \tag{34}$$

i.e. in the prograde direction, and the nodes moves with the velocity

$$\omega_{\theta} = \omega_{K} - \omega_{z} < 0 \tag{35}$$

in the retrograde direction.

Further we obtain from (10), (34), and (35)

$$\omega_{\pi} + \omega_{\theta} = \frac{\omega_{\pi}^2 + \omega_{\theta}^2}{2\omega_{K}}.$$
 (36)

As the right-hand term is very small we find in a first approximation

$$\omega_{\pi} = -\omega_{\theta} \tag{37}$$

which is a well-known result in celestial mechanics.

Introducing (37) into (36) we get in a second approximation

$$\Delta\omega = \omega_{\pi} + \omega_{\theta} = \frac{\omega_{\pi}^2}{\omega_{\kappa}}.$$
 (38)

Brouwer (1945) has treated the case when a satellite moves so close to a rotating central body that the quadrupole term of the equatorial bulge becomes important, and applied the result to the motion of Amalthea (Jupiter V), for which we have from observations

$$\omega_K = 722 \text{ deg. day}^{-1} = 722 \cdot 365 \text{ deg. year}^{-1};$$
 $\omega_{\pi} = 917.4 \text{ deg. year}^{-1}; \quad \omega_{\theta} = -914.7 \text{ deg. year}^{-1}.$
(39)

From an analysis according to the usual methods of celestial mechanics, Brouwer finds

$$\Delta\omega = 3.2 \text{ deg. year}^{-1} \tag{40}$$

(in reasonable agreement with the observational value 2.7 deg.).

Our formula (38) gives

$$\Delta\omega = \frac{917^2}{722 \cdot 365} = 3.16 \text{ deg. year}^{-1}.$$
 (41)

Hence for this case our simple formula seems to give a result in good agreement with the elaborate treatment by the usual methods.

7. Planetary Motion Perturbed by Other Planets

The motion of the body we are considering is perturbed by other bodies orbiting in the same system. Except when the motions are commensurable so that resonance effects become important, the main perturbation can be computed from the average potential produced by the other bodies.

As most satellites are very small compared to their central bodies, the effects described in Section 6 dominate in the satellite systems. On the other hand the perturbation of the planetary orbits is almost exclusively due to the gravitational force of the planets, among which the gravitational effect of Jupiter dominates. In order to calculate this one smears out Jupiter's mass along its orbit and computes the gravitational potential from this massive ring. It produces a perturbation which both outside and inside Jupiter's orbit obeys (32). Hence (33), (34) and (35) are also valid.

The dominating term for the calculation of the perturbation of the Jovian orbit derives from a similar effect produced mainly by Saturn.

8. Effect of Viscosity

Suppose that two particles with masses m_1 and m_2 move in orbits with angular momenta C_1 and C_2 ($C_2 > C_1$). Around circles with radii r_1 and r_2 they perform oscillations with amplitudes r_1e_1 , r_1i_1 ; and r_2e_2 , r_2i_2 , respectively.

If r_1 $(1+e_1) \ge r_2(1-e_2)$ they have a chance of colliding. If the precession rates of their nodes and perihelia are different and not commensurable, they will sooner or later collide at a point at the central distance r_3 . At the collision their tangential velocities will be $v_1 = C_1/r_3$ and $v_2 = C_2/r_3$. If the collision is perfectly inelastic, their common tangential velocity v_3 after the collision will be $v_3 = (m_1v_1 + m_2v_2)/(m_1 + m_2)$. Hence each of them will have the angular momentum

$$C_3 = \frac{m_1 C_1 + m_2 C_2}{m_1 + m_2} \,, \tag{42}$$

which means $C_1 < C_3 < C_2$. Collisions which are at least partially inelastic will tend to equalize the angular momenta of colliding particles. It is easily seen that collisions also will tend to make the particles oscillate with the same amplitude and phase. This means that the general result of viscosity is to make the orbits of particles more similar. In other words, the effect of viscosity is to produce what we may call an apparent attraction between the orbits.

9. Jet Streams

Suppose that a body moves with velocity v_0 and semi-major axis r_0 in a Kepler orbit which is sufficiently close to a circle to allow us to treat it according to Sections 3-8. Suppose further that the field is an unperturbed Coulomb field. Hence the orbit of the body will remain an ellipse which does not precess.

Let the body emit particles in all directions with the velocity v. The particles emitted in the radial direction will oscillate around the orbit of the body with the amplitude

$$x = r_0 v / v_0 \,. \tag{43}$$

Particles emitted in the axial direction will oscillate with the same amplitude. Further, a particle ejected in the tangential (forward) direction will have the angular momentum $C=r_0(v_0+v)$ which because $C=\sqrt{\mu r}$ is the same as a particle oscillating around the circle

$$r' = \frac{(r_0 v_0)^2}{\mu} \left(1 + \frac{v}{v_0} \right)^2 \approx r_0 \left(1 + \frac{2v}{v_0} \right). \tag{44}$$

Hence it will oscillate with the amplitude 2x and its maximum distance from the orbit of the body is 4x.

The particles emitted with a velocity v will remain inside a torus with the small radius $= x' = \beta x$ where x is given by (43) and β is between 1 and 4 depending on the angle of emission. This result also applies to the case when the body does not move exactly in a circle.

If a body, or a number of bodies in the same orbit, emit gas molecules with an rms thermal velocity $v = (3kT/m)^{1/2}$, the gas will be confined within a torus with the typical thickness of 4x, with

$$x = vr_0/v_0 = (3kTr_0^3/m\mu)^{1/2}. (45)$$

As a typical example, the thermal velocity of hydrogen molecules at T=300 K is of the order 10^5 cm/sec . If we put $v_0=3\cdot 10^6 \text{ cm/sec}$ (= the Earth's orbital velocity), we have $x/r_0=1/30$.

Hence the body, or bodies, may be surrounded by an atmosphere. The gravitation which prohibits the evaporation of the atmosphere of, say, the Earth, may in effect be substituted for by the forces f_r and f_z from (6) and (8) (although there is not a perfect similarity). The outer parts of the torus move slower, the inner parts swifter than the body in the central orbit.

The rings we consider differ from the rings in Laplacian theories. In our rings the mean free path is long compared to the dimensions. Further, our rings need not necessarily be circular. In fact, the phenomena we are discussing will take place even if the rings are ellipses with high eccentricity.

Suppose that there is a *jet stream*, consisting of a large number N of particles all confined to move inside a torus with small radius = x. In relation to particles moving in the central orbit their velocity is v. If all particles have the same mass m and the same collisional cross-section σ , each particle in the torus will collide with a frequency which is of the order

$$\frac{1}{T_{v}} = v\sigma n \tag{46}$$

where T_{ν} is the average time between two collisions and

$$n = \frac{N}{2\pi^2 r_0 x^2} \tag{47}$$

is the density. If each particle is a sphere with radius r and average density θ , its mass is $m = 4\pi\theta r^3/3 = 4\theta\sigma r/3$. The density of the grains may be about 3 g cm⁻³; but because the shape of the grains usually deviates so much from the spherical form (they may often be needles), we may for the order of magnitude put $\theta = \frac{3}{4}$ g cm⁻³; then $m = \sigma r$. If we put the space density $\varrho = m \cdot n$ we have

$$T_{v} = \frac{r}{vo} \,. \tag{48}$$

If we consider $v = 10^5$ cm sec⁻¹ as a typical relative velocity we find the values for $T_{\nu}\rho$ given in Table I.

TABLE I $r = 10^{-3}$ 1 10^{3} 10^{6} cm $T_{\nu}\varrho = 10^{-8}$ 10^{-5} 10^{-2} 10^{+1} sec g cm⁻³ $\varrho = 3 \cdot 10^{-21}$ $3 \cdot 10^{-18}$ $3 \cdot 10^{-15}$ $3 \cdot 10^{-12}$ g cm⁻³ $n = 10^{-12}$ 10^{-18} 10^{-24} 10^{-30} cm⁻³

In order to keep the jet stream together the collisional time T_{ν} must be smaller than the time constant for the disruptive processes. Most important of these are the differential precession of the different orbits in the jet stream, and the Poynting-Robertson effect. For the order of magnitude we may put $T_{\nu} = 10^5$ years = $3 \cdot 10^{12}$ sec (cf. Whipple, 1968). This value gives the values of ϱ in Table I.

The contraction of a jet stream is produced by inelastic collisions between the particles. The time constant for contraction should be a few times T.

This also means that a jet stream can be formed only when there is no disruptional effect with a shorter time constant. For example, the differential precession of the pericentre and the nodes of an elliptic orbit will disrupt a jet stream, unless T_{ν} is smaller than the period of the differential precession.

A more refined model should take account of the size distribution of the particles. Since the smallest particles are usually the most numerous, these will be the most efficient in keeping a jet stream together.

When the relative velocities in the interior of a jet stream decrease, the accretion of grains to larger bodies will be more and more efficient. Hence for the order of magnitude T_{ν} is a sort of *coagulation* time of the jet stream. However, if larger bodies are formed the result is that T_{ν} will increase and the contractional force will be smaller. Eventually the jet stream may no longer keep together.

10. Collisions between a Grain and a Jet Stream

Stars, planets, nebulae, galaxies, and the like are usually called 'celestial objects'. Jet

streams of the type we have considered deserve to be classified as a new type of celestial object.

We shall study what happens if a grain with mass m_g collides with a jet stream. Suppose that the grain moves in an orbit which at one point P crosses the jet stream. See Figure 2. (In principle its orbit could cross the jet stream at four points, but we confine ourselves to the simplest case.) We are considering motions in an unperturbed Coulomb field, which means that the orbits remain unchanged unless the particle collide.

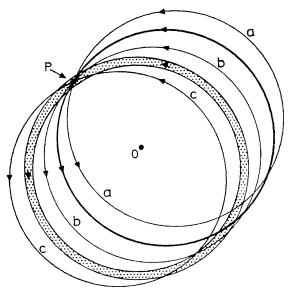


Fig. 2. Capture of a grain by a jet stream. The shaded area represents the jet stream. The orbit of a grain (thick curve) intersects the jet stream at P. Collisions lead to fragmentation and fragments are ejected for example along the thin curves. All these orbits carry them back to the point P. Subsequent collisions at P may lead to further fragmentation, but if the collisions are at least partially inelastic the final result is that all the fragments will be captured by the jet stream.

In the region where the grain crosses the jet stream, it will sooner or later collide with one of the particles of the jet stream. The collision is supposed to be partially inelastic, which means that part of the kinetic energy due to the relative motion is dissipated. The collision may result in a splitting of one of the colliding grains, or of both, into a number of fragments.

After the collision each of these fragments will move in a new orbit which in general differs from the initial orbits of the grains. The orbit may be situated inside the jet stream, but it may very well be outside the jet stream. However, all possible orbits of the fragments will necessarily bring them back again to the point where the collision took place. As by definition this was situated inside the jet stream, all the fragments will again and again cross the jet stream. (An exception to this rule occurs when the collision has taken place near the surface of the jet stream and the latter has had time to contract so much before the next collision that the point of the first collision is outside the stream.) Sooner or later this orbital intersection will lead to new collisions with the particles in the jet stream,

As on the average the collisions smooth out the relative velocities, the fragments will finally be captured by the jet stream. At the same time this capture will change the shape of the jet stream so that the new orbit is a compromise between its original orbit and the orbit of the colliding grain.

Hence a grain which collides with a jet stream will be 'eaten up' by it, either at once or after having been fragmented. In the latter case the jet stream 'chews' before it 'swallows'.

In this way new kinetic energy is transferred to the jet stream so that the decrease in its internal energy may be compensated. If a large number of grains are colliding with the jet stream, a temporary state of equilibrium is attained when the losses due to internal collisions are compensated by the energy brought in by the 'eaten' particles. However, the new particles increase the value of N, and hence the losses. The final destiny is in any case a collapse of the jet stream.

The internal structure of a jet stream depends on the size distribution and on the velocity distribution. We have only discussed the ideal case when all particles are identical. In a real jet stream there is likely to be an assortment of particles of all sizes, some accreting to larger particles, and some being disrupted by collisions. When the internal energy of the jet stream decreases, the relative velocities will be smaller. This means that the collisions will not so often lead to disruption. The accretion will dominate, and larger bodies will be formed inside the jet stream.

If the Coulomb field is perturbed, the jet stream will precess, the nodes moving in the retrograde and the pericentre in the prograde sense. However, the rate of precession depends on the orbital elements; and these are slightly different for the particles inside the jet. Hence the perturbations tend to disrupt the jet stream. The permanence of the jet stream depends upon whether the viscosity, which keeps the jet stream together, is strong enough to dominate. In general, large bodies will leave the jet stream more readily than small bodies.

We conclude:

- (1) If a large number of grains are moving in a certain region near a central body, jet streams may be formed. These are kept together by viscosity (mutual collisions).
 - (2) The jet streams have a tendency to capture all grains which collide with them.
 - (3) The jet streams have a tendency to contract.
 - (4) Inside a jet stream the grains will aggregate to larger bodies.
 - (5) Large bodies formed in a jet stream may break loose from it.

11. The Importance of Jet Streams

Jet streams may be of importance in three different connections.

A. METEOR SHOWERS

A meteor shower is usually considered to be fragments of a disrupted comet. There is a clear association between some comets and meteor streams, but there is no convincing argument for the view that all meteor streams are disrupted comets. This

is claimed only because no other mechanism for meteor stream formation has been suggested.

As we have found that a meteor stream can keep together because of its internal viscosity, it is not necessary to postulate that all meteor streams are disrupted comets. We may as well assume that meteor streams are produced by the 'viscosity attraction' we have studied. Inside a jet stream the grains have a tendency to accrete and this may even lead to the formation of comets.

Hence in principle comets may be produced by jet streams.

This does not necessarily mean that jet streams cannot be produced by comets. Comets and meteors may be in a sort of equilibrium with grains which sometimes accrete to comets, sometimes spread out to meteor streams.

Although this picture seems to be of interest in a qualitative way, it is not quite clear whether any present-day meteor streams have sufficient density for the contraction mechanism to work.

B. ASTEROIDAL JET STREAMS

As mentioned in Section 1 there are jet streams among the Hirayama families. They may in principle be formed by the 'viscosity attraction'. Inside the streams an accretion to larger asteroids may take place.

Those Hirayama families where no jet streams have been found may still contain jet streams consisting of small bodies. They could also represent the end result of the action of a jet stream.

The contraction must essentially be due to the action of small asteroids and asteroidal grains. As the density of these is unknown, we cannot decide whether the jet stream theory is applicable. This decision is a challenging question for space research in connection with the exploration of the asteroid belt.

C. JET STREAMS AND THE COSMOGONIC PROCESS

The condensation from a partially corotating plasma produces a great number of grains moving in eccentric orbits $(e=\frac{1}{3})$. It is possible, perhaps even likely, that the viscosity attraction collects these grains into jet streams, which then capture more and more grains.

If this view is correct, there is a complicated series of processes by which grains form jet streams, the streams capture more grains, and bigger bodies are built up inside the jets. The net result is that the eccentricity and inclination of the orbits of the initial grains decrease, for the oscillations in the radial and axial directions are damped. In the asteroid belt it is possible to study this process in some detail; it may be of fundamental importance to the understanding how planets are formed.

Anders (1965) and others have shown that the statistics of the size of asteroids may be interpreted as consisting of two parts, one identified with the 'original condensation' and the other a result of collisions. It is possible that the two parts correspond to the build-up process inside the jets and the splitting which occurs when asteroids collide mutually (or with jet streams).

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ABSTRACTS OF FORTHCOMING PAPERS

Beatrice M. Tinsley: Comments on 'On Old Universes and Galaxy Evolution' by Chambers and Roeder. (Received 17 July, 1969.)

Chambers and Roeder estimated the effect of galactic evolution on the magnitude-redshift relation by calculating the change in 3×10^9 years of an empirical model elliptical galaxy. They found that the evolutionary effect alone was too small to reconcile the observations with model universes of low density and age greater than that of globular star clusters. It is shown here that because these authors made theoretical errors in calculating both the luminosity of the past galaxy and its effect on the observed apparent magnitude, they greatly underestimated the evolutionary corrections. Revised estimates of the corrections are given here, using the same model galaxies as Chambers and Roeder as far as possible, and imposing as they do the further constraints that the luminosity function of the past galaxy be consistent with the present, and that the intrinsic color (B–V) change by less than $\sim 3\%$ out to redshift z=0.2. The revised corrections at z=0.46 are possibly great enough to allow low-density cosmological models older than 13×10^9 years if Hubble's constant $H_0=100$ km sec⁻¹ Mpc⁻¹, and are certainly great enough to allow such models older than 10×10^9 years if $H_0=75$. With any reasonable past model of the empirical galaxy, evolution raises the observed luminosity at $z\sim 0.5$ in any model universe by the significant amount of several tenths of a magnitude.

E. R. Mustel and A. A. Boyarchuk: Structure of Envelopes Ejected by Novae. (Received 25 July, 1969.)

The paper contains an analysis of the structure of envelopes ejected during the outbursts of Novae. The data used for this purpose: (a) Direct photographs of envelopes and the photographs taken with the use of different colour filters; (b) Spectra of envelopes. The envelope of DQ Her is studied most carefully. The analysis of all available data for the envelopes around DQ Her and V 603 Aql permits to outline a morphological model of these envelopes. It appears, that the structure of both these envelopes is approximately identical and that the difference in the observed properties of the photographic images of the nebulae is connected with a difference in the orientation of the 'polar' axis of the envelopes relatively to the line of sight. The envelope ejected during the outburst of T Aur reveals the same properties, which are characteristic for the envelopes of DQ Her and V 603 Aql.

From this we conclude that the distribution of gases inside the envelopes of the majority of Novae is approximately of the same character. This speaks in favour of the presence of certain forces around many Novae, which guide the motion of ejected plasma along some quite definite directions inside rather small solid angles. It seems that the only conceivable forces of this type may be the forces of a magnetic nature. This hypothesis for example permits to explain the difference between the envelope of GK Per and the envelopes of DQ Her, V 603 Aql, T Aur.

Comparing the velocity of expansion of the envelope of DQ Her and the rate of change of its angular size we computed that the distance to DQ Her is equal to 320 pc.

On the base of photographs of the envelope of DQ Her it is found that in 1968 the flux $F_{\text{H}\alpha}$ of radiation in H α -line was equal to $(6 \pm 2) \times 10^{-12}$ ergs/cm² sec, whereas the mass of the envelope was equal to 10^{29} g and its electronic concentration n_e to 2×10^3 cm⁻³. Several hypotheses, which may explain the stratification of emission from different elements inside the envelope are discussed.

R. D. Weidelt: Stellar Wind for Non-Negligible Gravitational Potential of the Atmosphere. (Received 28 July, 1969.)

We consider a spherically symmetric, isothermal and stationary stellar atmosphere whose gravitational potential cannot be neglected. If κ , the ratio of the density on the base of the corona to the mean

density of the star is not zero, the density vanishes at infinity for any solution of the hydrodynamic equations. We deduce the maximum mass loss depending on two dimensionless parameters, the ratio of the gravitational to the thermal energy and κ . This mass loss has itself a maximum for $\kappa = 1/3$.

W. D. Langer and L. C. Rosen: Hyperonic Equation of Space. (Received 4 August, 1969.)

An equation of state for cold matter at neutron star densities, $\varrho > 10^{14}$ gm/cm³, is evaluated. The gas is considered to be a degenerate mixture of neutrons, protons, leptons, hyperons and massive baryons. We derive the equilibrium equations including the effects of nuclear interactions among all the hadrons.

Jeffrey M. Cohen, William D. Langer, Leonard C. Rosen, and A. G. W. Cameron: Neutron Star Models Based on an Improved Equation of State. (Received 4 August, 1969.)

Using an equation of state for cold degenerate matter which takes nuclear forces and nuclear clustering into account, neutron star models are constructed. Stable models were obtained in the mass range above $0.065~M_{\odot}$ and density range $10^{14.08}$ to $10^{15.4}~\rm gm/cm^3$. All of these models were found to be bound. The outer crystaline layer of the star was found to have a thickness of 500 m or more depending on the mass of the model.

Guilio Calami: The Stability of the Equilibrium of Supermassive and Superdense Stars. (Received 5 August, 1969.)

The second variation of the mass-energy (by constant baryon number) is analysed to inquire into the stability of the equilibrium for a spherical mass under the action of its own gravitation. The equations of general relativity are used. For zero density at the surface, the sphere is stable if (and only if) a solution of a second-order equation (Jacobi equation) has 'only' one zero in the interior (at center) of the sphere. The results obtained in this way coincide of course with those obtained by means of the Chandrasekhar-Misner-Zapolsky variational principle, but the solution of the Jacobi equation is more simple and straightforward. In particular, the stability of the several modes are analysed without making any choice of a trial function for the Lagrangian displacement.

The method is applied to the Gratton R-polytropic model of supermassive stars.

D. K. Sarvajna: Orbits of Charged Bodies. (Received 7 August, 1969.)

An investigation into the possibility that material drawn out of a star goes into orbit around that star if electromagnetic effects are included, has been made. It is found that if the body has an initial charge of some 10³⁷ e.s.u., and decreasing with time then sufficient angular momentum can be transferred to make orbits not intersecting the stellar surface possible.

Jeffrey M. Cohen: General Relativistic Rotational Properties of Stellar Models. (Received 11 August, 1969.)

In this paper we give general relativistic expressions for the angular momentum and rotational kinetic energy of slowly rotating stars. These expressions contain contributions from the pressure, gravitational red shift, and Doppler shift, and the motion of inertial frames. These contributions are not negligible, e.g., there are stable neutron star models for which the angular velocity of inertial frames at the center is about 70% the angular velocity of the star. These expressions are useful in the study of pulsars if pulsars are rotating neutron stars.

D. Sher: Potential Energy of Continuous and Discrete Distributions of Matter from the Point of View of the Application of the Virial Theorem. (Received 12 August, 1969.)

The potential energy of clusters of stars in which the distribution of matter is taken to be continuous is compared with that of static model clusters in which the distribution of matter is discrete, the comparison being made from the point of view of applying the virial theorem to estimate the masses of the clusters.

There is good agreement on the average between the two cases as long as the stellar distribution is random. Systematic differences occur whenever there is any departure from randomness. However, reduction of the mass of a cluster as estimated by means of the virial theorem by even as much as a factor of 2 on the average would seem to require even greater departures from randomness in the stellar distribution than are considered here. As might be expected there are sometimes very large fluctuations in the potential energy from one cluster to the next in the discrete case.

I. Lerche: Effect of Interstellar Density Fluctuations on Signal Dispersion Measure. (Received 8 September, 1969.)

The effect of electron number density fluctuations in the interstellar medium on signals from pulsars is studied in terms of the frequency dependent signal dispersion. It is shown that if the density fluctuations are representative of long wavelength ($1 \sim 100 \, \mathrm{pc}$) [or large scale length ($1 \sim 100 \, \mathrm{pc}$)] disturbances in the interstellar gas, then the observed signal dispersion is not a measure of the integral of the electron number density in the line of sight. Evidence has been presented elsewhere for believing that such long wavelength disturbances should exist in the interstellar gas, so this result indicates that some care must be exercised in the interpretation of signal dispersion measurements from pulsars.

A. Kovetz: Maximal Red Shifts of Neutron Stars. (Received 8 September, 1969.)

It has been recently established that there exists a maximal red shift z_{max} for a homogeneous star of given mass M. The relationship z_{max} (M) is obtained for neutron stars in the mass range $0.71 \le M/M_{\odot} \le 12.06$.

Hideyuki Niimi: Stability of the System of Stars, Gas and Magnetic Fields. (Received in revised form, 11 September 1969.)

Linear stability of a system of stars, gas and magnetic fields under the existence of a relative motion between the stars and the gas is investigated by the use of the magnetohydrodynamic and the polytropic equations for the gas and the collisionless Boltzmann equation for the stars together with the Poisson equation. The star system is supposed to have the anisotropic Schwarzschild distribution. The critical wavenumber is calculated and it is found that the system becomes universally unstable under some conditions.

M. N. Rao: Rare Gases on the Earth and in Meteorites. (Received 15 September, 1969.)

Analysis of abundance patterns of rare gases Ne²², Ar³⁶, Kr⁸⁴ and Xe¹³⁰ on Earth and in ordinary and carbonaceous chondrites is presented. A mechanism of chemical adsorption of rare gases at the planetesimal stage during their accretion is proposed to generate the abundance pattern of the heavy rare gases on the Earth. The calculated values for Xe and Kr agree well with the observed values whereas for Ar, the agreement is poor.

Terry W. Edwards: Perturbation Theory for Stellar Interior Problems. (Received 22 September, 1969.)

A rather general first order linearized perturbation theory is derived with which one can calculate a quasi-static equilibrium stellar interior model if given an initial approximation thereto. The process developed is applicable to both static and evolutionary models, wherein slightly different physical assumption (e.g. gas characteristics) or physical conditions (e.g. matter composition) are imposed. The correction to the given model is obtained through bi-directional quadrature with fitting at an intermediate position; however, no trial integrations are necessary.

Siddheshwar Lal and Karl Brunstein: **Homogeneous Metagalactic Origin of Cosmic Rays**. (Received 29 September, 1969.)

Starting with the hypothesis that cosmic rays are evenly distributed in the metagalaxy, it is shown that the flux of the electron-positron component, which is produced through π - μ -e decays, following the nuclear collisions of the cosmic-ray beam with the intergalactic medium, takes $\leq 4 \times 10^{16}$ sec to reach steady-state. The corresponding value of the flux of the *positron component* and its implications regarding the homogeneous model of the metagalactic origin of cosmic rays are discussed.

Hans-Christoph Thomas: A Polytropic Model for the Helium Shell Flash. (Received 29 September, 1969.)

A model consisting of two polytropes is constructed, to represent a helium core of a star during the helium shell flash occurring at the onset of helium burning in a degenerate core. The maximum temperature reached during the flash can be predicted as a function of core mass and mass inside the helium burning shell. This temperature will generally be too low for the production of neutrons out of ^{14}N . Some additional results on the helium shell flash in a star of 1.3 M_0 are also presented.

Donald B. Melrose: On the Isotropization of Electrons in Synchrotron Sources. (Received 30 September, 1969.)

Electrons radiating synchrotron radiation develop a pitch angle anisotropy, and so become unstable to the coherent emission of hydromagnetic waves. The evolution of the coupled system of anisotropic electrons and waves is studied in the absence of any dissipation of the waves in the ambient medium. The anisotropy of the electrons approaches a steady state in which the anisotropy is energy independent and of order v_A/c (v_A = Alfvén speed). The conditions for this small degree of anisotropy to be maintained are examined.

Due to this scattering the bend in the synchrotron spectrum, from an inverse power law with index α to one with index $\frac{4}{3}\alpha + 1$, due to an initial or recurrent injection of electrons, could only occur at infrared or higher frequencies.

F. W. Stecker: The Cosmic Gamma-Ray Spectrum from Secondary Particle Production in Cosmic-Ray Interactions. (Received 30 September, 1969.)

The cosmic γ -ray spectrum below 1 GeV arising from cosmic ray p-p interactions is calculated. Its characteristics are determined by the properties of secondary neutral pion production occurring at accelerator energies. A model is chosen for numerical calculations in which the two dominant modes of neutral pion production at accelerator energies are the production of the $\Delta(1.238)$ isobar and one fireball. The effect of α -p and p- α interactions on the cosmic γ -ray spectrum is also calculated. The final results are given in terms of both differential and integral γ -ray energy spectra.

R. Cowsik, Yash Pal, and T. N. Rengarajan: A Search for a Consistent Model for Electromagnetic Spectrum of the Crab Nebula. (Received 30 September, 1969.)

An attempt is made to search for a consistent model to explain the electromagnetic spectrum of the Crab nebula (Tau A). It is assumed that there is a continuous injection of electrons at the centre of the nebula with an energy spectrum $E^{-1.54}$ as evidenced by radio data. This spectrum must stepen to a slope larger than 2 at some energy E_i in order to ensure that the energy input into electrons remains finite. The spectrum must also steepen beyond an energy E_c depending on the magnetic field because of synchrotron energy losses. Two types of models are considered: Class I, in which the whole nebula is characterised by a uniform magnetic field and Class II in which besides the general field H_0 , small filamentary regions of strong field H_s are postulated.

In models of Class I, the best fit to the observed data is obtained when $E_i > E_c$ and $H_0 \simeq 5 \times 10^{-4}$ gauss. However, this predicts a decrease in X-ray source size beyond ~ 40 KeV. There are two possibilities of Class II model depending on the residence time of electrons in strong field regions being small or large. The former case explains the flattening in the optical spectrum.

Experiments to distinguish between the various models are indicated.

A. Kovetz and G. Shaviv: Maximal Masses of Fe⁵⁶ and Ni⁵⁶ Stars. (Received 1 October, 1969.)

Upper bounds are derived for the masses of pure Ni⁵⁶ and Fe⁵⁶ stars, which are dynamically stable with respect to photodisintegration. Possible effects on stellar evolution are discussed.

Kris Davidson: The Development of a Cocoon Star. (Received 20 October, 1969.)

A newly formed massive star is likely to be surrounded by dense gas and dust as it approaches the main sequence. Radiation pressure must push some of the inner material outward before the star begins to produce ionizong radiation; this affects the formation of the HII region. A remarkably dense 'dust front' may precede the ionization front.

The observable radio and infrared spectra are discussed. If the dust cloud is composed of small graphite grains, extraordinarily large far-infrared fluxes are possible.

R. A. Sunyaev and Ya. B. Zeldovich: Small-Scale Fluctuations of Relic Radiation. (Both in Russian and English) (Received 2 November, 1969.)

Perturbations of the matter density in a homogeneous and isotropic cosmological model which leads to the formation of galaxies should, at later stages of evolution, cause spatial fluctuations of relic radiation. Silk assumed that an adiabatic connection existed between the density perturbations at the moment of recombination of the initial plasma and fluctuations of the observed temperature of radiation $\delta T/T = \delta \varrho_m/3 \varrho_m$. It is shown in this article that such a simple connection is not applicable due to:

- (1) the long time of recombination;
- (2) the fact that when regions with $M < 10^{15} M_{\odot}$ become transparent for radiation, the optical depth to the observer is still due to Thompson scattering;
 - (3) the spasmodic increase of $\delta \varrho_m/\varrho_m$ in recombination.

As a result the expected temperature fluctuations of relic radiation should be smaller than adiabatic fluctuations. In this article the value of $\delta T/T$ arising from scattering of radiation on moving electrons is calculated; the velocity field is generated by adiabatic or entropy density perturbations. Fluctuations of the relic radiation due to secondary heating of the intergalactic gas are also estimated. A detailed investigation of the spectrum of fluctuations may, in principle, lead to an understanding of the nature of initial density perturbations since a distinct periodic dependence of the spectral density of perturbations on wavelength (mass) is peculiar to adiabatic perturbations. Practical observations are quite difficult due to the smallness of the effects and the presence of fluctuations connected with discrete sources of radio emission.