A THREE-RING CIRCUIT MODEL OF THE MAGNETOSPHERE

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Abstract. We have modeled the magnetosphere by superimposing a dipole field, a uniform field and a perturbation field due to a simple current system. This current system consists of a ring current in the neutral line of the dipole plus uniform fields, together with vertical currents representing field-aligned currents to the neutral line. The current circuit is closed by two additional ring currents above and below the equatorial plane representing distributed adiabatic perpendicular currents. This system produces many magnetospheric features including a magnetopause, bending of magnetic field lines in the anti-solar direction, a magnetotail, and cusps on the day-side of the Earth. Our aim is to demonstrate that it is not necessary to think of the magnetic field topology as being caused by the flowing plasma carrying field lines. The fundamental physical problem is to derive the current system from the self-consistent interaction of the solar-wind and magnetospheric plasmas and fields.

1. Introduction

The shape of the magnetopause and the topology of the magnetic field lines in the interaction regions between the solar wind plasma and the magnetosphere are conventionally described in terms of magnetohydrodynamics. In this description, the shape of the magnetopause and the flow pattern of the plasma are determined by a dynamic balance between the pressure of the geomagnetic field and its plasma, and the pressure of the impacting solar wind and magnetic field. The magnetic field lines are picturesquely thought of as being carried along with the plasma flow and draped around the magnetospheric cavity. This picture is intuitively appealing and appears to have been quite successful in explaining such things as the existence, shape, and position of the sunward magnetopause (e.g., Parker, 1969).

However, there is a more fundamental viewpoint based on the fact that any magnetic field configuration is ultimately determined by the volume distribution of currents. The currents, in turn, are determined by the distribution of particles and their velocities. The problem is to solve in a self-consistent manner for both the particle motions and the resultant electric and magnetic fields (Whipple, 1979). Needless to say, this is an extremely complicated problem which is not yet near solution. In the magnetohydrodynamic approach, the current aspect is neglected and not usually explicitly displayed, although for the most part it is implicitly included in the formulation. The most important current system as far as the magnetopause configuration is concerned, however, is the thin current sheet existing at the magnetopause itself. Since the magnetopause layer has a thickness on the order of the ion gyroradius, it is not possible within the

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constraints of conventional magnetohydrodynamics to derive this current system in terms of the adjacent plasmas. Thus the concept of field line 'merging' or 'reconnection' has arisen to describe what is essentially the deviation from classical magnetohydrodynamic behavior in these thin boundary layer regions.

The purpose of this paper is to demonstrate that an extremely simple current system can reproduce some of the important topological features of the Earth's magnetosphere. We do not address the problem of how these currents are obtained in a self-consistent manner from the interaction of the solar wind and magnetospheric plasmas, but instead we start with a simple model and hope that successive approximations will lead to a consistent description. Our main point, however, is that it is not necessary to think of the magnetic field topology in the boundary regions of the magnetosphere as being caused by the flowing plasma carrying along the field lines. Rather, it is more fundamental to think of the current system and how the magnetic field due to the current system superimposes on the already-present dipole and uniform fields of the Earth and undisturbed solar wind.

2. Kinds of Currents

Recently, Alfvén (1977, 1979a, b, 1981) has discussed the importance of describing plasma phenomena in space in terms of the plasma currents. Some advantages of such a formulation are:

(1) The circuit of the current demonstrates the importance of boundary conditions which are often neglected.

(2) By directing attention to the electromotive forces driving the current, and to the regions of dissipation, the energy transfer from one region to another is more easily understood.

(3) Certain types of important current-produced phenomena, including the formation of double layers, are difficult to understand without accounting for the current explicitly.

Three main types of electric currents are of importance:

(1) Field-aligned currents parallel to the magnetic field (Birkeland currents).

(2) Currents along neutral lines or in surfaces where the magnetic field is small enough that the adiabatic approximation for the particle motion breaks down.

(3) Adiabatic drift currents (including diamagnetic currents) which are primarily perpendicular to the magnetic field.

A theoretical understanding of the magnetic field configuration in the magnetosphere, or in any system involving the interaction of a plasma with a magnetic field, involves a formulation of the plasma currents in terms of the physical processes driving the currents, and then a calculation of the magnetic field perturbation using Maxwell's equation relating the field to the currents.

As an example of this kind of formulation, Alfvén (1979) considered the magnetosphere as a dipole field (M = dipole moment) representing the terrestrial field superimposed by a homogeneous interplanetary field $\mathbf{B}_s$, and a uniform solar wind with velocity $\mathbf{v}_s$ perpendicular to the dipole axis. Hence, the magnetosphere is immersed in an electric field which at large distances from the Earth has the constant value $\mathbf{E}_s = \mathbf{v}_s \times \mathbf{B}_s$. Figure 1 shows the resultant magnetic field configuration when the interplanetary magnetic field is directed southward. This configuration has the following properties:

(1) A neutral line ($B = 0$) is produced in the equatorial plane at a distance R from the dipole center, with $R = (M/B_s)^{1/3}$. This is defined as the limit of the magnetosphere.

(2) The field lines through the neutral line asymptotically approach a cylinder with a radius $r_s = R\sqrt{3}$ at large distances from the dipole.

(3) The homogeneous electric field E_s at infinity causes a voltage difference $V = 2r_sE_s$ between the points a and d, and between a' and d' in Figure 2. The magnetic field configuration in the zero-order approximation where the field lines are considered to be equipotentials transfers this potential drop to the magnetosphere. This produces a sunward plasma drift in the equatorial region of the

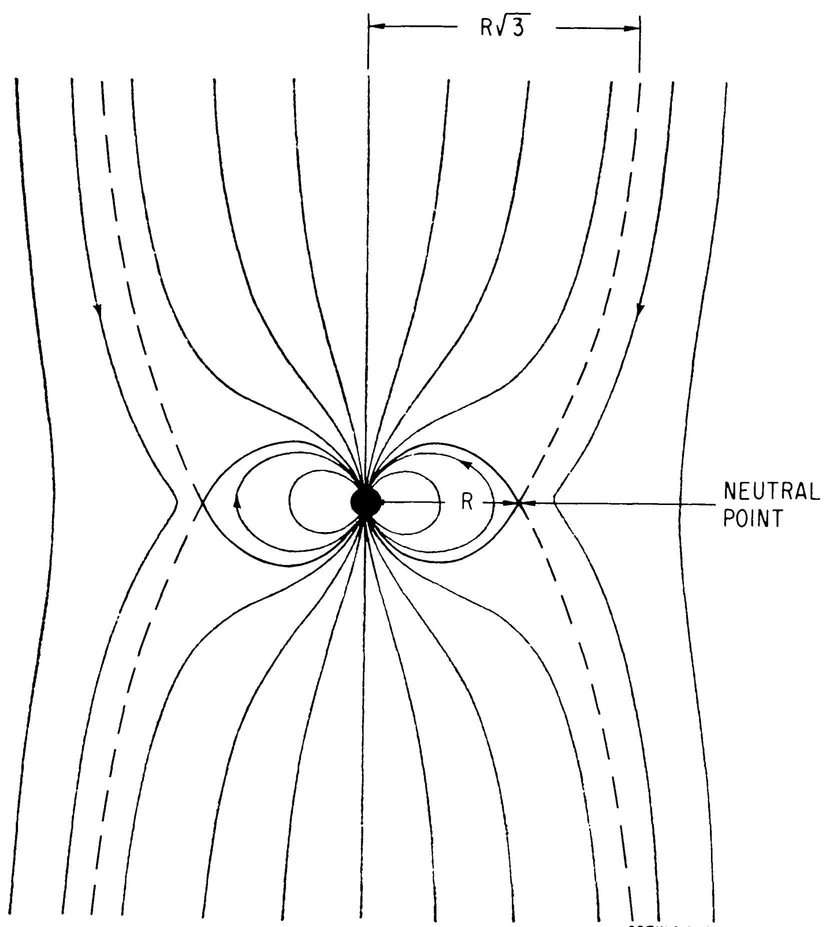

Fig. 1. Magnetic field configuration due to a dipole plus southward interplanetary field.

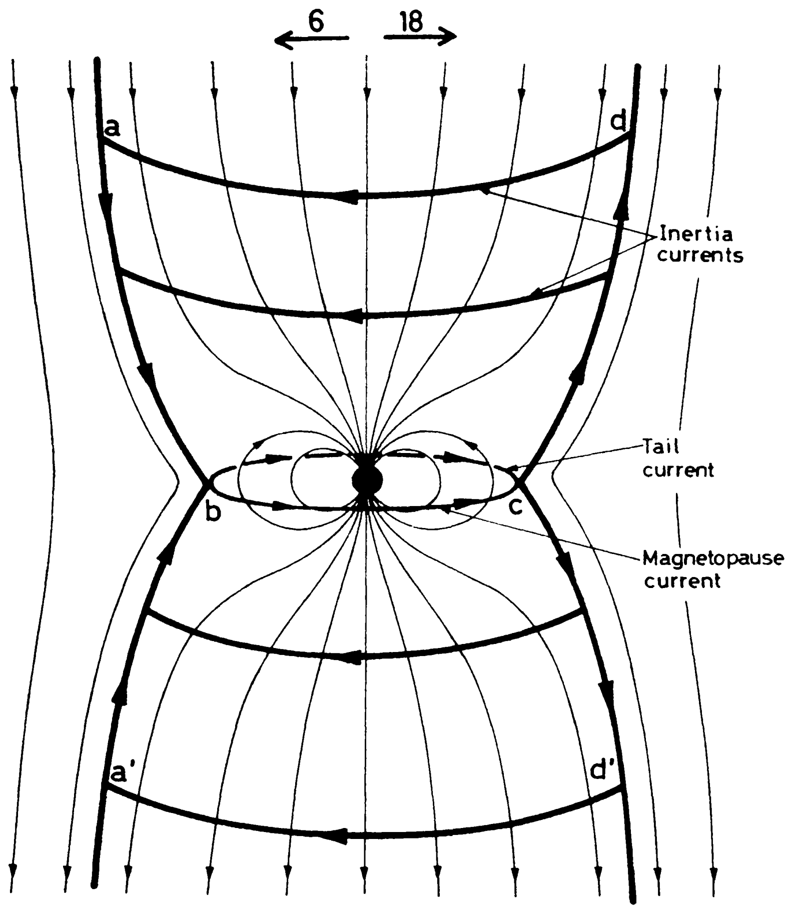

Fig. 2. Current system seen from Sun (from Alfvén, 1979a). The solar wind electric field produces a voltage difference between the field lines *aba'* and *dcd'*, which produces a current along the sunward and the antisolar parts of the neutral line bc. It also produces field-aligned and perpendicular currents over the magnetosphere.

magnetosphere. It also produces a dawn-to-dusk current along the sunward and anti-sunward sectors of the neutral line. The magnitude of the current along the neutral line is assumed to be proportional to the component of the electric field tangent to the neutral line direction. Since the electric field is directed from dawn to dusk, the tangential electric field and hence the current magnitude will vary as ($-\cos\Phi$), where Φ is the angle corresponding to the local time (i.e. $\Phi = 0$ at local midnight).

(4) Since the current in the neutral line varies with azimuthal position, there must be field-aligned and/or perpendicular currents to satisfy the current continuity condition. Field-aligned currents are especially important on those field lines connecting to the neutral line. Perpendicular currents are distributed throughout the interaction region and are due to magnetization effects, curvature and gradient drifts, and inertial drift. The latter is probably especially significant because it taps kinetic energy from the deceleration of the solar wind and converts it into magnetic energy through the formation of currents (Alfvén and Fälthammar, 1963).

The first-order current system in this configuration consists of the neutral line current together with currents in the surface defined by the field lines going through the neutral line, as shown in Figure 2. These latter currents close the circuit and consist of both field-aligned components and also distributed transverse currents due to inertial effects. The effect of this current system in perturbing the magnetic field configuration is shown qualitatively in Figure 3. On the day side, the perturbation field from the current system adds to the vacuum field to produce a feature similar to the magnetopause. On the night side, the perturbation field opposes the vacuum field to produce a tail-like behavior.

The following sections of this paper describe quantitative results for a

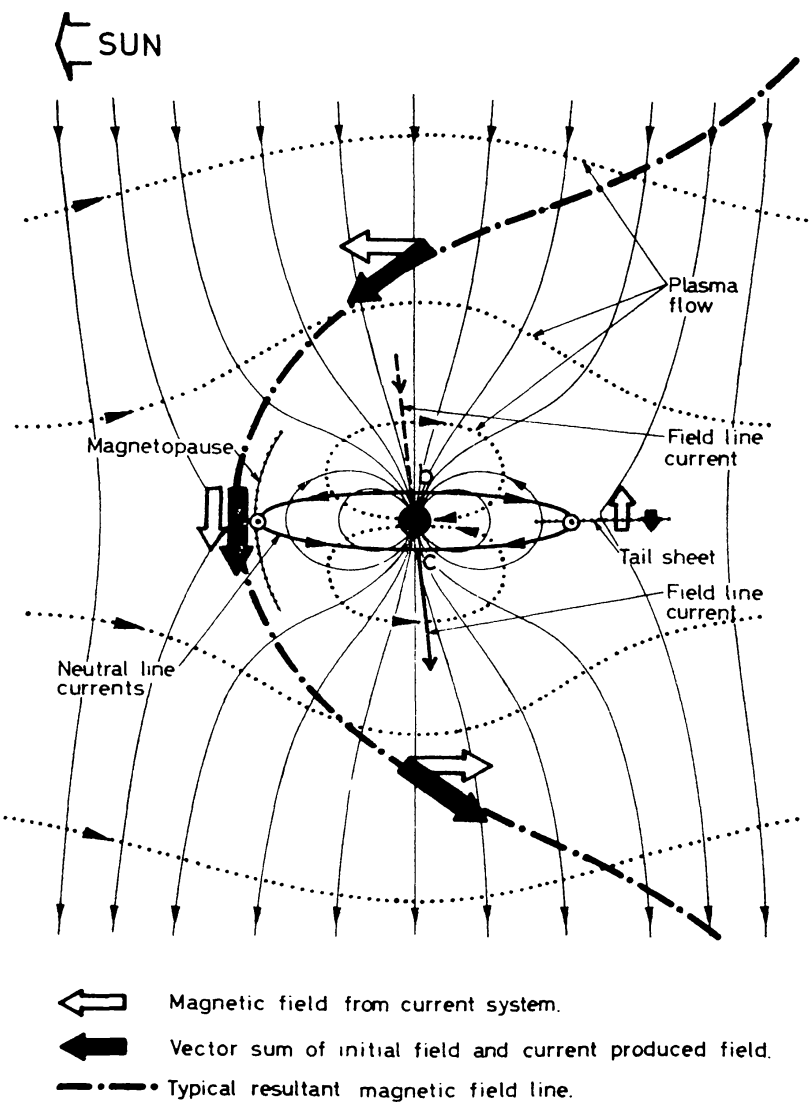

Fig. 3. Current system seen from dusk side (from Alfvén, 1979b). This also shows the plasma flow and the resultant magnetic field configuration. The field aligned currents to the dawn and dusk sides of the neutral line are closed through the sunward part of the neutral line (resulting in the formation of the magnetopause) and the antisolar part (resulting in the tail neutral sheet).

simplified version of this first-order current system. In this simplified version, the distributed currents in the surface consisting of the field lines going through the neutral line are replaced by vertical currents in a cylindrical surface going through the neutral line, together with two additional ring currents at remote distances above and below the equatorial plane to close the current system. These two additional ring currents represent the distributed perpendicular currents in the interaction region. In spite of these rather drastic simplifications, it will be shown that large-scale features such as the shape and position of the magnetopause and the extension of a magnetotail behind the Earth are obtained with this model.

3. The Three-Ring Circuit

Figure 4 shows the three-ring circuit approximation to the current system of Figure 2. The azimuthal current density in the equatorial neutral line at $r = R$ is given by

$$J_\Phi = -2I_0 \cos \Phi \delta(r - R)\delta(z) . \tag{1}$$

The vertical currents above and below the neutral line are determined by the conditions of current continuity and symmetry. They are

$$J_z = \pm (I_0/R) \sin \Phi \delta(r - R) , \tag{2}$$

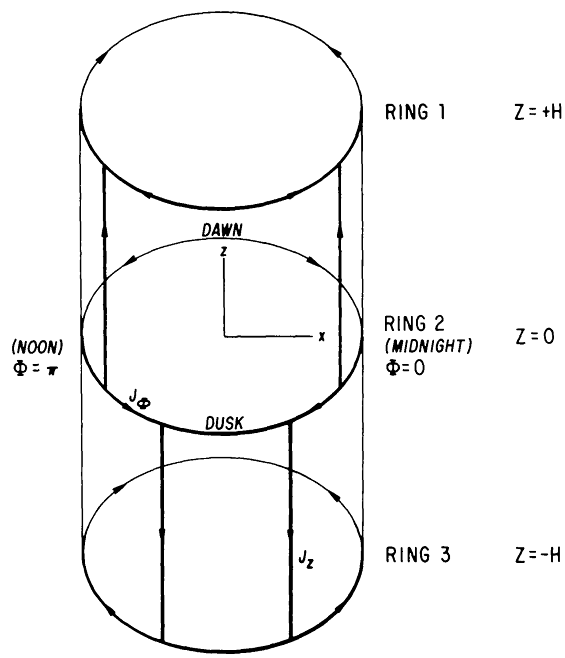

Fig. 4. The three-ring circuit model. The center ring represents the current in the neutral line. The vertical currents in the cylinder sheet represent field-aligned currents to the neutral line. The other two ring currents above and below the equatorial plane close the current system and represent distributed perpendicular currents.

where the − sign refers to the currents above the equatorial plane, and the + sign refers to currents below the equatorial plane. To complete the circuit two additional ring currents are present at heights $z = \pm H$ above and below the equatorial plane. These rings represent the distributed horizontal currents outside of the equatorial plane. In reality, the horizontal currents probably fall off gradually with distance from the equatorial plane. In our model we place these two current rings well above and below the region of interest so that the magnetic field topology in the interaction region is determined primarily by the neutral line ring current and the associated vertical currents. The ring current densities at $z = \pm H$ are given by

$$J_\Phi = I_0 \cos \Phi \delta(r - R)\delta(z \pm H) . \tag{3}$$

The current parameter I_0 in these equations can be chosen arbitrarily, although we will interpret its magnitude as being proportional to the interplanetary electric field $\mathbf{E}_s$.

The magnetic field configuration due to this current system is obtained by the usual method of finding the vector potential and then differentiating to get the magnetic field components. The results for the Cartesian components of the magnetic field vector in MKS units are

$$B_x = -K\sum_{i=1}^{3} I_i(z - z_i)\left[\cos^2 \Phi(G_i + M_i) + \sin^2 \Phi\left(E_i + J_i - \frac{r}{R} H_i\right)\right], \tag{4}$$

$$B_y = -K \sin \Phi \cos \Phi \sum_{i=1}^{3} I_i(z - z_i)\left[G_i + M_i - E_i - J_i + \frac{r}{R} H_i\right], \tag{5}$$

$$B_z = K \cos \Phi \sum_{i=1}^{3} I_i[rG_i - RF_i] . \tag{6}$$

In these equations the constant $K = \mu_0 R/4\pi$, and the quantities I_i and z_i are given by $I_1 = I_3 = I_0$, $I_2 = -2I_0$; $z_1 = H$, $z_2 = 0$, $z_3 = -H$. The other quantities are functions of the radius, r, and are essentially elliptic integrals (our notation), defined as

$$\begin{aligned} E_i &= \int_0^{2\pi} \frac{\sin^2 \alpha \, d\alpha}{[C_i - D \cos \alpha]^{3/2}}, & J_i &= \int_0^{2\pi} \frac{\cos^2 \alpha \, d\alpha}{C_0[C_i - D \cos \alpha]^{1/2}}, \\ F_i &= \int_0^{2\pi} \frac{\cos \alpha \, d\alpha}{[C_i - D \cos \alpha]^{3/2}}, & H_i &= \int_0^{2\pi} \frac{\cos \alpha \, d\alpha}{C_0[C_i - D \cos \alpha]^{1/2}}, \\ G_i &= \int_0^{2\pi} \frac{\cos^2 \alpha \, d\alpha}{[C_i - D \cos \alpha]^{3/2}}, & M_i &= \int_0^{2\pi} \frac{\sin^2 \alpha \, d\alpha}{C_0[C_i - D \cos \alpha]^{1/2}}, \end{aligned} \tag{7}$$

where

$$\begin{aligned} C_0 &= r^2 + R^2 - 2rR \cos \alpha, \\ C_i &= r^2 + R^2 + (z - z_i)^2, \\ D &= 2rR . \end{aligned} \tag{8}$$

The total magnetic field is then the superposition of the field given by Equations (4) through (6) with the dipole and uniform field configuration shown in Figure 1. The magnetic field configurations shown in the following sections were obtained by numerical step-by-step integration along the field lines, with the direction cosines given by the ratios of the field components to the total magnitude. The integrals in (7) were evaluated by using a Gaussian quadrature technique.

4. Results

Figures 5 through 7 show the resulting magnetic field configurations for three values of the current I_0. The figures are in the meridian plane going through local noon and midnight, with the x-axis positive to the right. The Sun may be thought of as being to the extreme left with the solar wind plasma flowing from left to right. In each of these figures the dipole moment of the Earth is 31000 (gamma $\times R_E^3$) where R_E is the Earth radius, and the interplanetary field is 10 gamma. The three values of the current I_0 are 5, 15, and 45 times the quantity (4π gamma $* R_E/\mu_0 = 63710$ amp). Thus the currents are respectively, 3.2×10^5, 9.6×10^5, and 2.9×10^6 ampères. The neutral line position for these parameters is

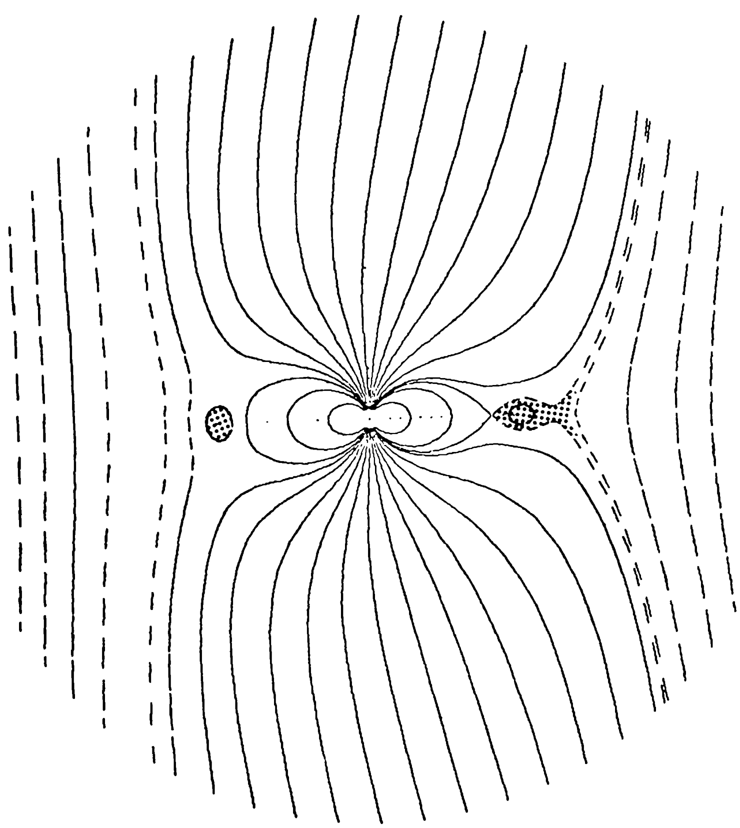

Fig. 5. Magnetic field configuration for a current value (I_0) of 3.2×10^5 amp. The shaded areas are artifacts due to neutral lines instead of neutral surfaces representing the magnetopause and the tail neutral sheet.

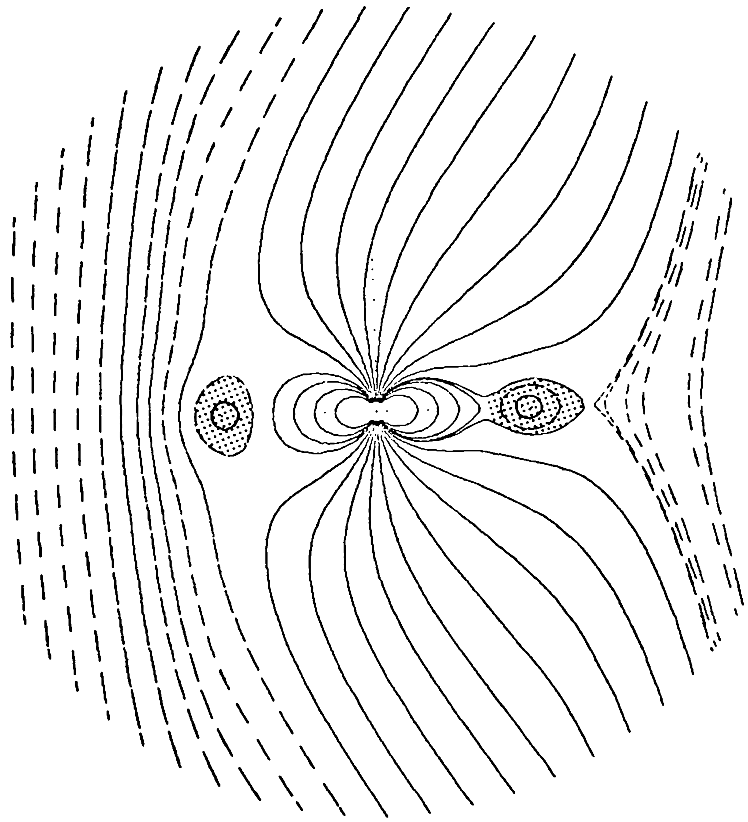

Fig. 6. Magnetic field configuration for current value (I_0) of 9.6×10^5 amp. The Sun is to the left, with the solar wind flowing from left to right.

$R = 14.581\ R_E$. The height of the other ring currents above and below the equatorial plane is $= \pm 70\ R_E$.

It is apparent from the figures how an increasing current progressively distorts the field lines from a relatively undisturbed configuration in Figure 5 (which is very similar to Figure 1) to a typical magnetospheric configuration. In Figure 7, the magnetopause may be taken, for example as the last open field line on the front side of the Earth. Both the open field lines and the field lines from the Earth's polar caps are bent backwards in the interaction regions towards the tail of the magnetosphere. In addition to the open field lines and field lines connecting to the Earth, there are closed field lines around the ring currents in this model. In the real magnetosphere, of course, these latter field lines are absent (or at least much weaker) since the actual currents are distributed rather than concentrated in a line.

The asymmetry between the front and tail regions of the magnetosphere in this model is due to the fact that on the front side, the field from the current (which is in the $-y$ direction, or out of the paper) reinforces both the interplanetary (southward) field outside the ring current and also the (northward) dipole field inside the ring current. On the tail side, the current is also in the $-y$ direction, and hence the field from the current opposes both the interplanetary

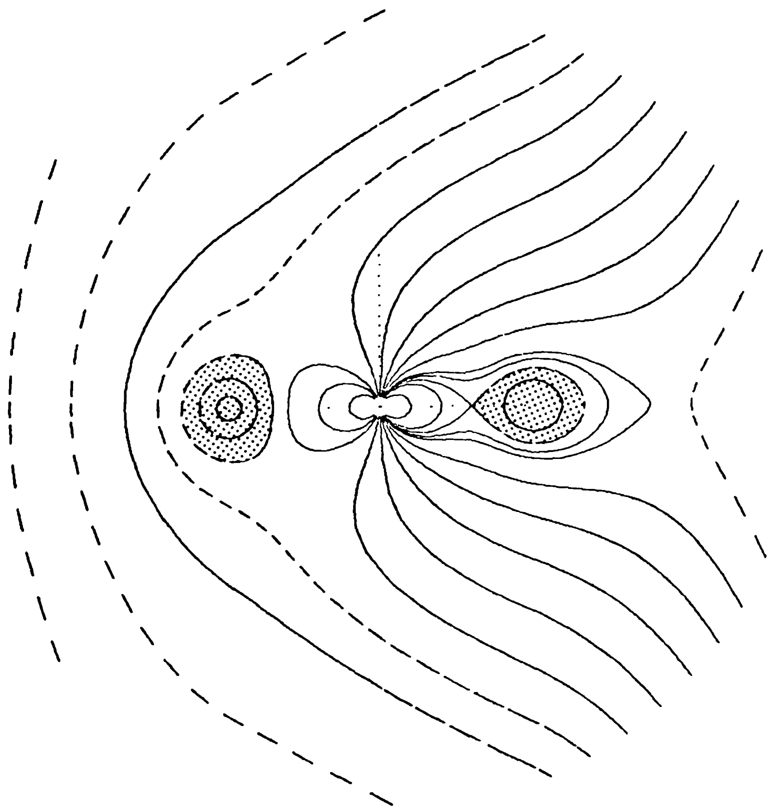

Fig. 7. Magnetic field configuration for Current value (I_0) of 2.9×10^6 amp. Note the bending of field lines in the anti-solar direction, the presence of cusps on the dayside of the earth, the existence of a magnetopause (first open field line outisde or to the left of the current), and the formation of a magnetic tail.

field on the outside and the dipole field on the inside. Thus two X-points are produced in the tail, one on each side of the ring current. One on the front side, two X-points are also produced, one above and one below the equatorial plane.

Figure 8 shows the shape of the magnetopause in the noon to dusk quadrant for the same current used in Figure 7. The field lines in the figure are the first open field lines (within $\pm 0.1\, R_E$) as one goes outward from the Earth in the equatorial plane. The magnetopause location is very nearly a circle at $20\, R_E$ in this quadrant for the parameter values chosen in this model. In the dusk to midnight quadrant the magnetopause is extended further, reaching $27\, R_E$ at midnight.

5. Discussion

By substituting the electric current description of the interaction between the solar wind and the magnetosphere for the magnetohydrodynamic description, we have obtained a procedure for calculating the magnetic field configuration. Of course, the current model that we have used in this paper is to a certain extent an ad hoc model, based on qualitative arguments. However, our aim is to demonstrate that it is not necessary to think of the magnetic field topology as being caused by the flowing plasma carrying field lines. The fundamental

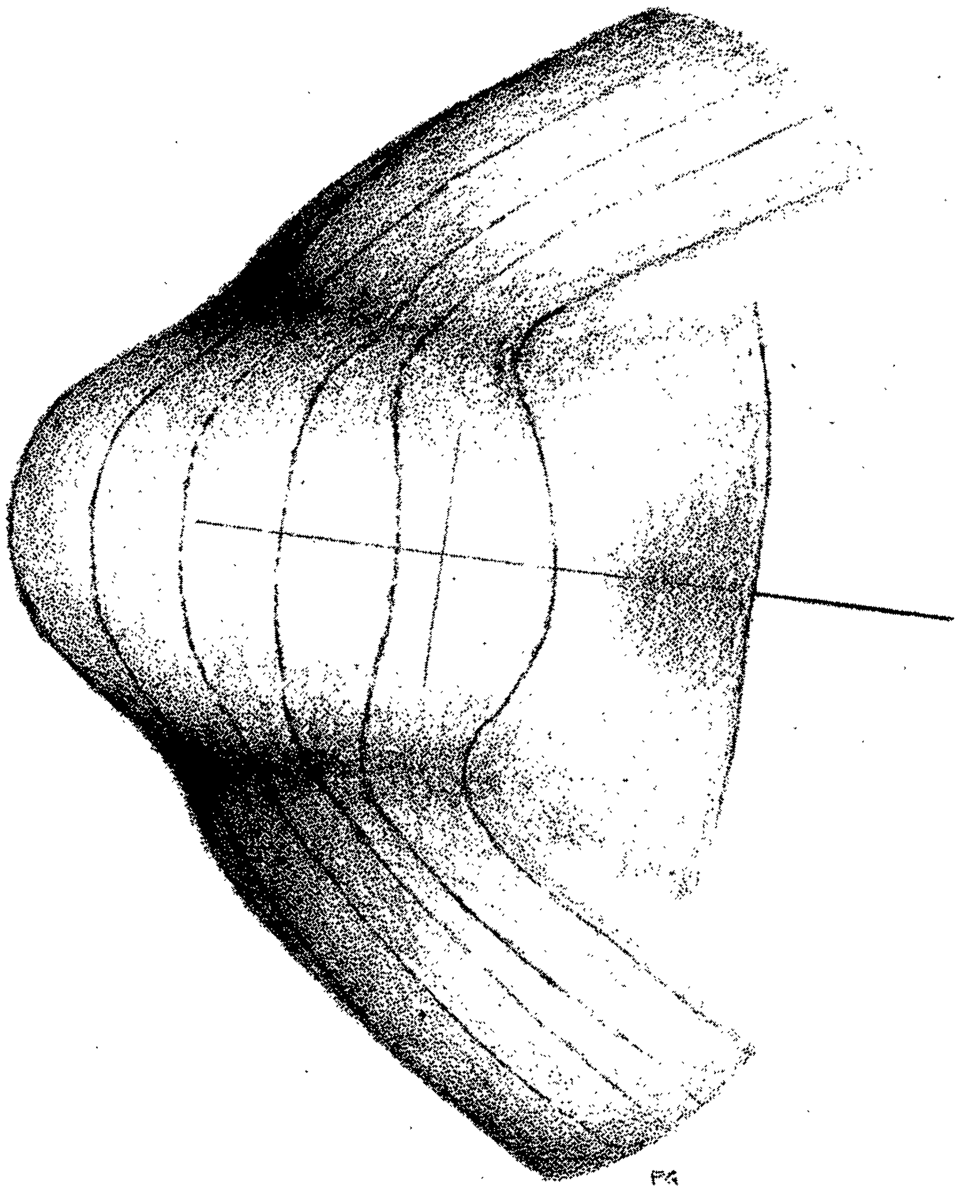

Fig. 8. The shape of the magnetopause in the noon to dusk quadrant. Magnetic field lines are shown every 15° in longitude from noon (180°) to dusk (270°). This surface is the set of first open field lines outside the equatorial ring current. The equatorial position of the magnetopause is at approximately 20 R_E for this model in the noon to dusk quadrant. In the dusk to midnight quadrant the magnetopause extends further, reaching 27 R_E at midnight. The surface between 255° and 270° is not shown at latitudes beyond about ±30° since it would partially obscure the surface behind it. The model is symmetric between the dawn and dusk sides of the Earth.

physical problem is to derive the current system from the self-consistent interaction of the solar-wind and magnetospheric plasmas and fields.

In spite of the crudeness of the circuit model that we have used, a number of the results are reconcilable with observations. These include:

(1) The bending of the magnetic field lines in the anti-solar direction.

(2) The existence of cusps on the day side of the Earth.

(3) The singular point on the dayside of our model separating into two singular points, one above and one below the equatorial plane, with the distance between them increasing with increasing current. This is formally an artifact of our model. It shows that in reality the neutral line current should be replaced by a surface current in a vertical surface representing the magnetopause.

(4) The singular point on the nightside also separating into two singular points as the current increases, with both of them in the equatorial plane. This shows in a similar way that the line current on the nightside should be replaced by a neutral sheet current in the tail.

(5) Other artifacts in our model are the closed field lines in the shaded regions of Figures 5 through 7 around the ring current, and also the distorted spiral shape of certain field lines in the vicinity of the dawn and dusk portion of the ring current.

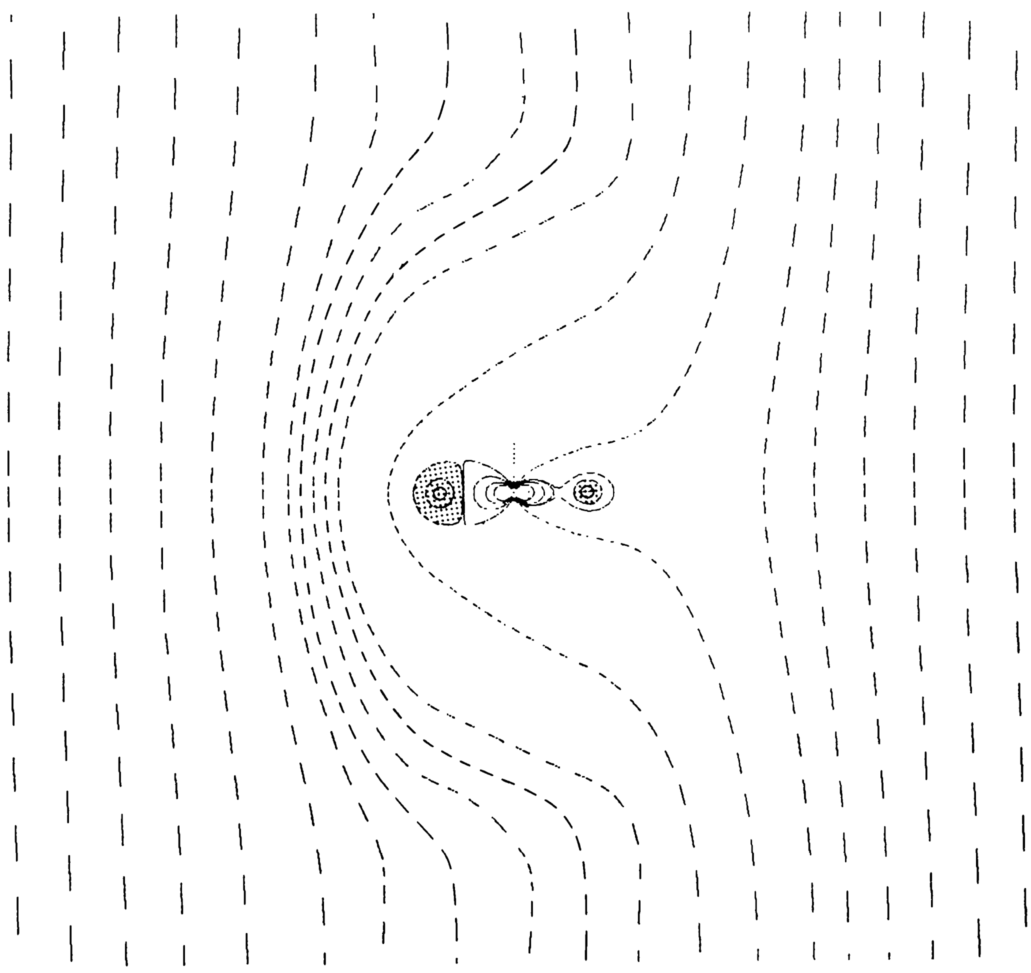

Fig. 9. Large scale view of the magnetic field configuration for the same case as Figure 7 ($I_o = 2.9 \times 10^6$ amp.). This shows how the magnetosphere is embedded in the solar wind magnetic field in this model, with all open field lines eventually ending up pointing in the north-south direction.

Figure 9 is a larger-scale view of the same configuration as in Figure 7 showing how the field lines connect back to the interplanetary field at large distances from the Earth. The distance at which this occurs and the shape of the field lines at these distances depend largely on the height of the additional ring currents above and below the Earth, and hence are also largely artifacts of the model. However, it seems to show that the representation of the interaction by a system of roughly circular inertial currents may still be valid when the magnetic field is highly distorted. In reality, these inertial currents are distributed in space rather than concentrated in a line.

The adiabatic currents in a plasma are well understood in the sense that these currents can be expressed in terms of the local properties of the plasma and magnetic field configuration. Hence the problem of including the adiabatic currents in an electric current model is largely mathematical: How can a computational scheme be set up which calculates the adiabatic currents self-consistently with the local magnetic field configuration? However, this is not the case with the field-aligned and neutral region currents: these currents are not well understood. Expressions for these currents are not available and it appears that magnetohydrodynamic considerations alone will not be sufficient to derive such expressions. We believe that this is the most critical area for research, at present, into the question of the configuration of the magnetosphere.

Acknowledgements

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